Geometry at the root of complexity

Sentinel North Scientific Meeting

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2023-11-15

Northern complex systems

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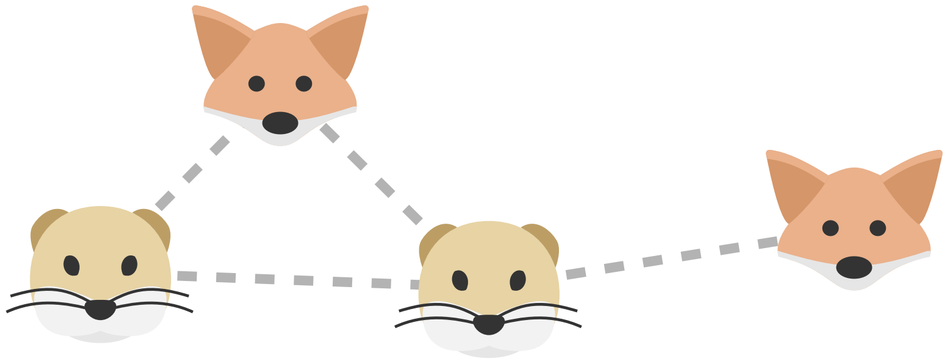
Northern research involves a large variety of complex systems such as

- brain networks;
- virus coocurence;
- ecosystems.



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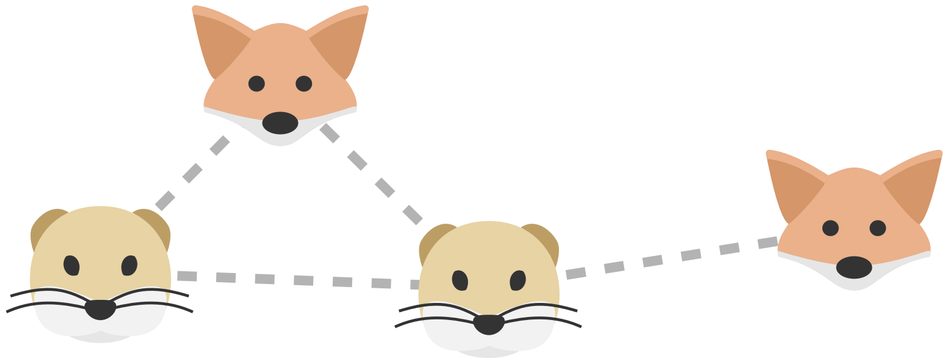
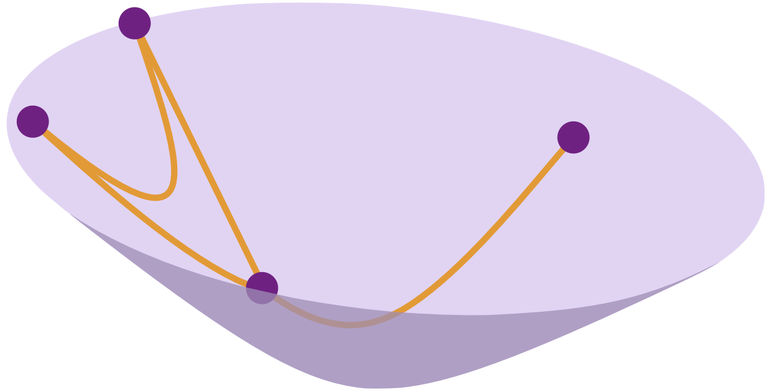


Perturbations due to stress **propagate** over the interaction network, whose **structure** greatly influences its capacity to **recover** from them (i.e. resilience).

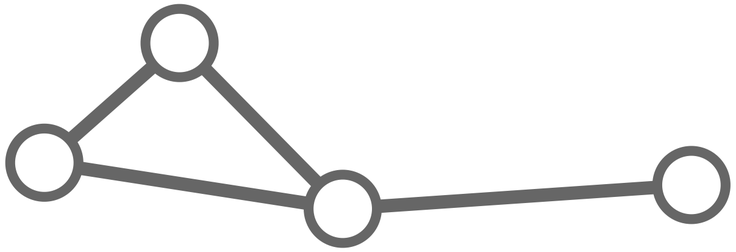
Abstracting the complex system's structure

Abstraction

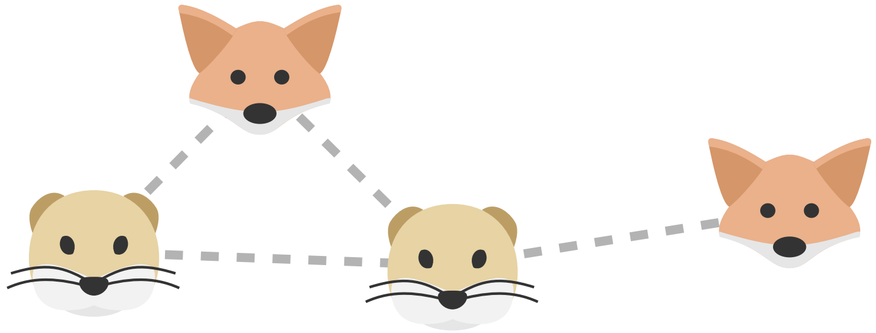
Layer 3
Embedding



Layer 2
Graph



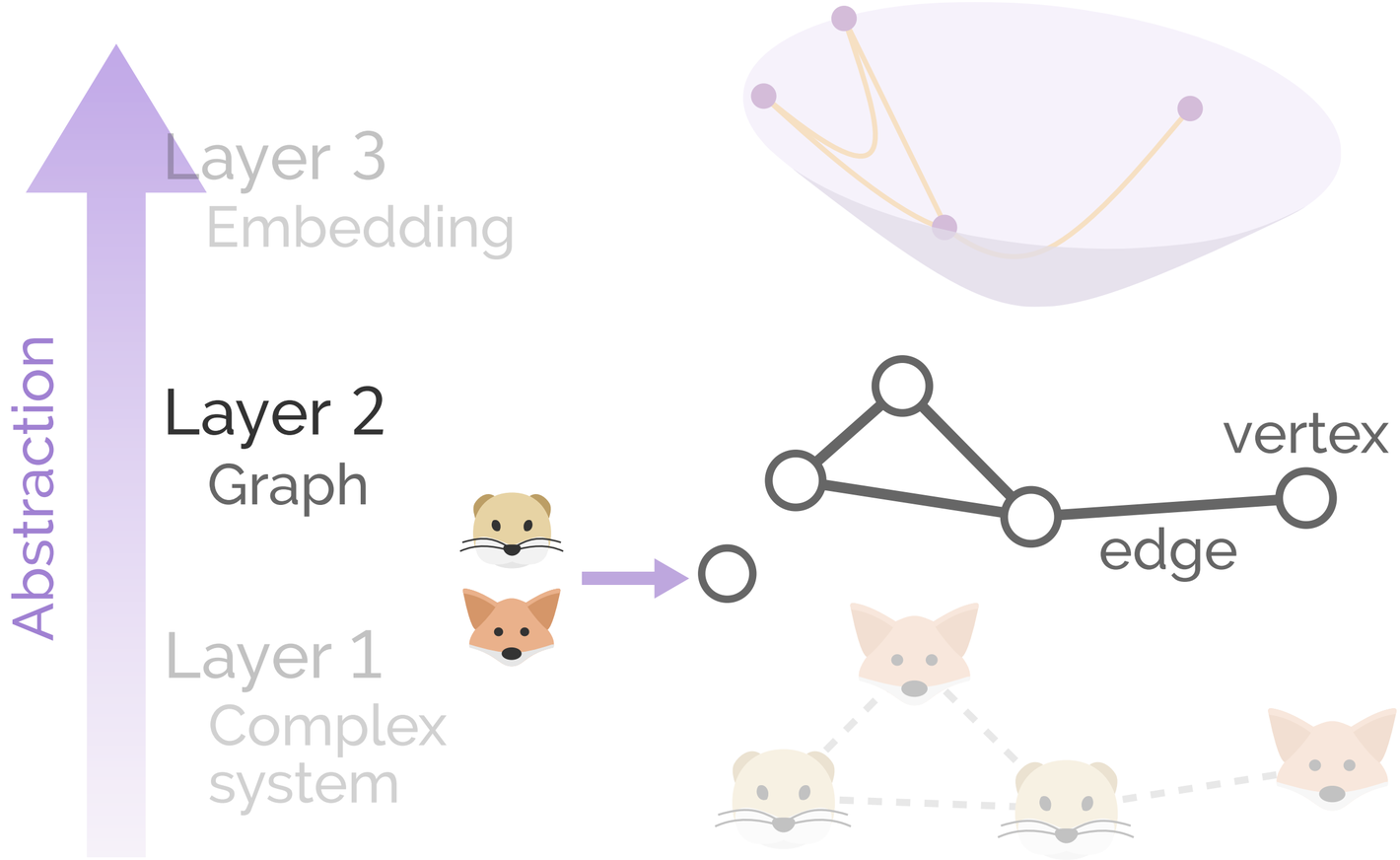
Layer 1
Complex system



Layer 2: graph

We suppose the behavior of a complex systems is governed by its interactions.

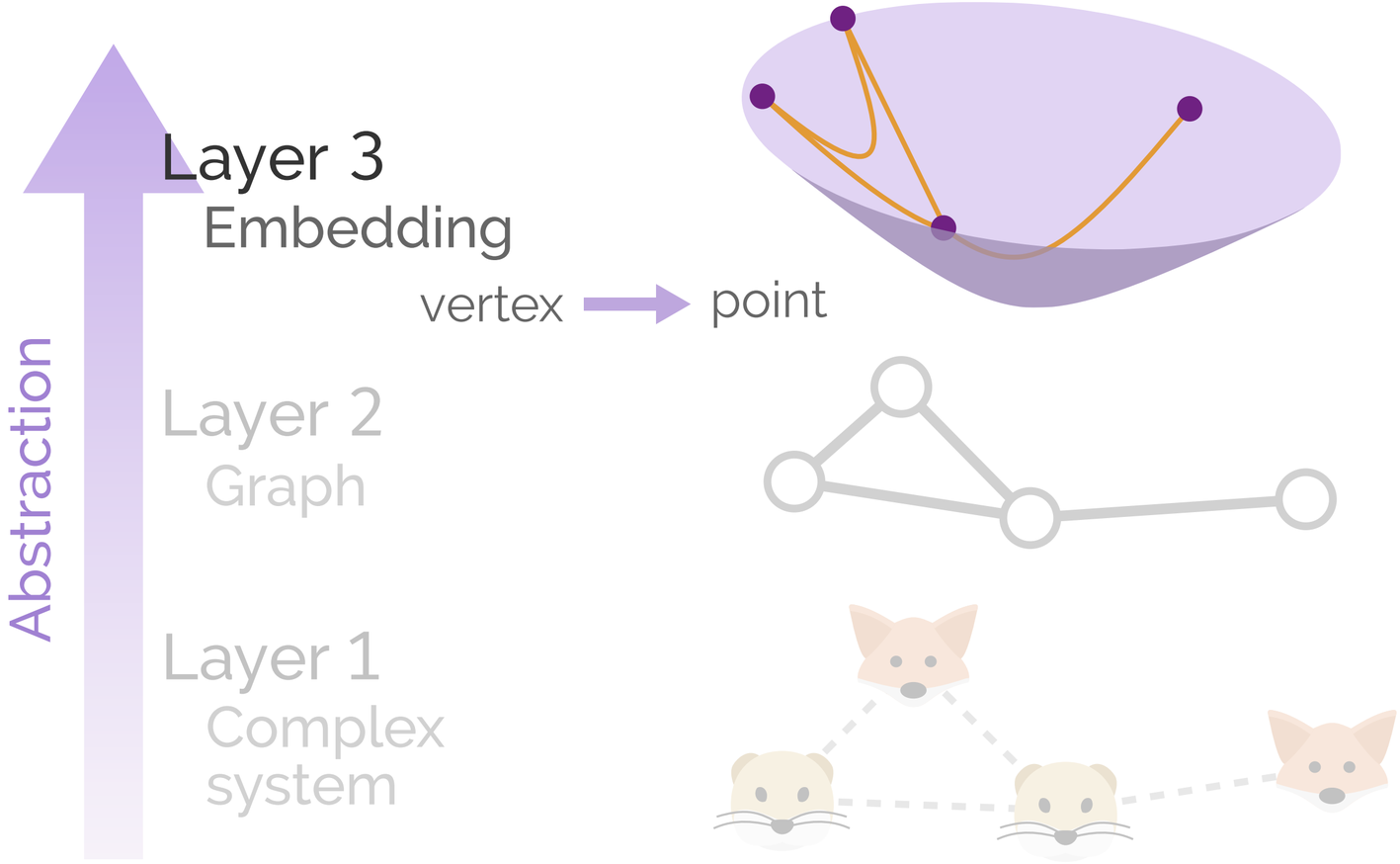
In a graph, the entities are **vertices** and their interactions are **edges**.



Layer 3: embedding

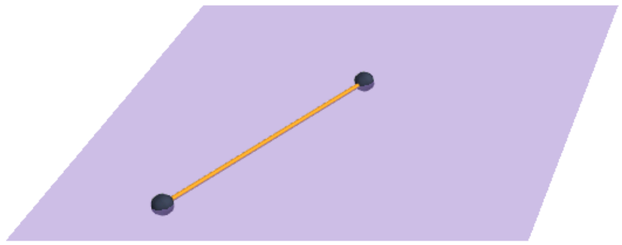
We suppose that vertices are points in a metric space. The positions form the **embedding** of the graph.

Edges are black and **shortest paths** (geodesics) are orange.

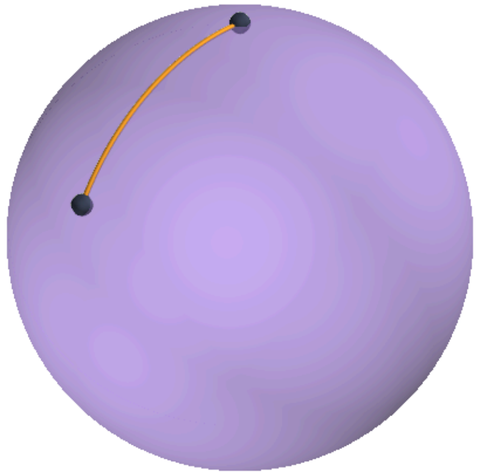


Shortest paths in metric spaces of constant curvature

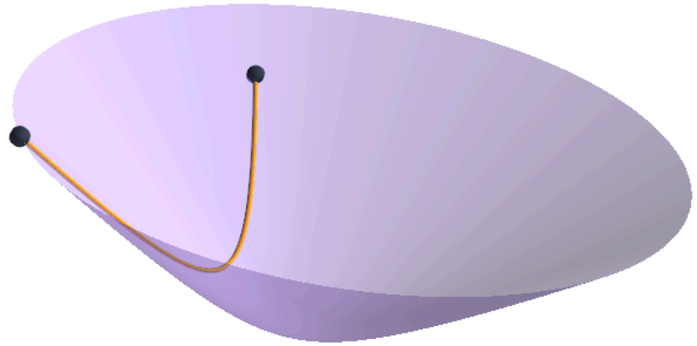
The shortest path between two points is curved except in the flat space.



Euclidean
No curvature



Spherical
Positive curvature

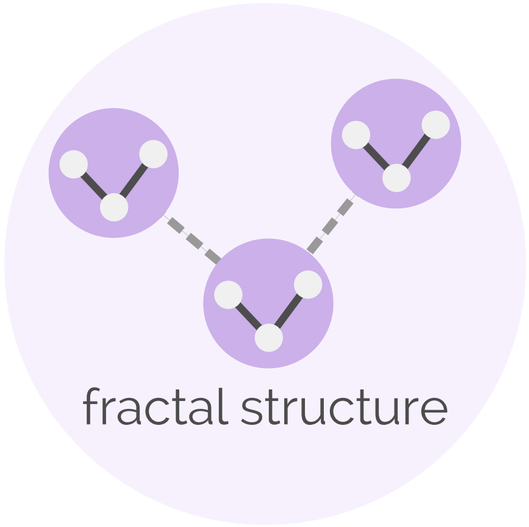


Hyperbolic
Negative curvature

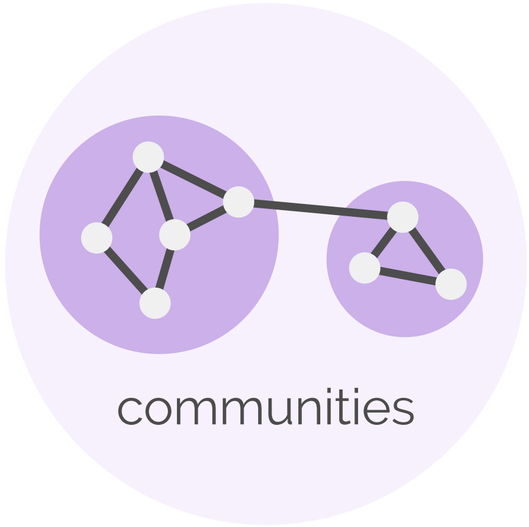
Hyperbolic geometry is ideal

We use the hyperbolic geometry because it naturally **reproduces important structural properties** observed in real complex system such as

- the fractal structure of many networks;
- the communities in the graph;
- the shortest paths.



fractal structure



communities

shortest paths similarity

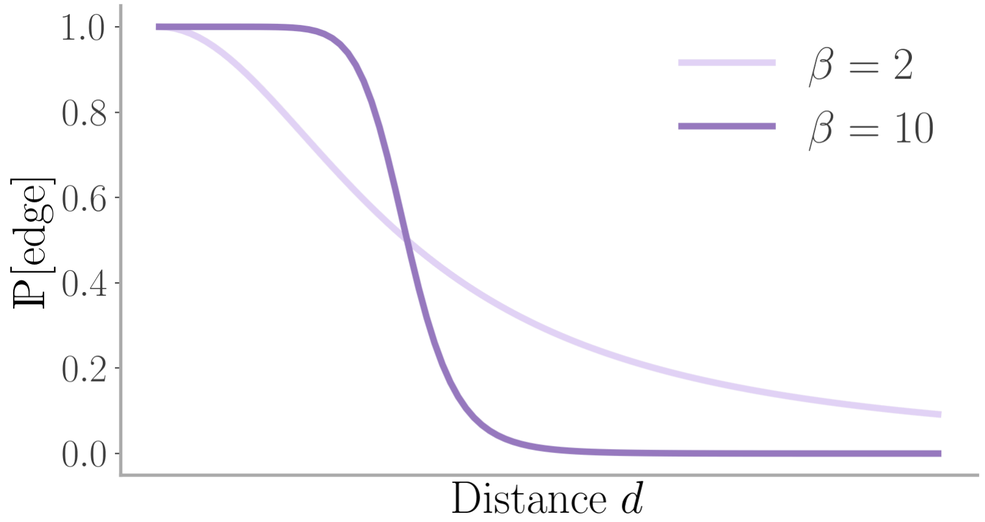
Probabilistic graph from the embedding

The embedding can be used **to generate** new graphs.

An edge exists with probability

$$\mathbb{P}[(i,j) \text{ is an edge}] = \frac{1}{1 + e^{\beta(d(x_i,x_j)-R)}},$$

where x_i is vertex i's position, d is the distance and R is the maximum radius.



Embedding is optimization

The embedding should generate the original graph with high probability, meaning that

- if (i, j) is an edge, $d(x_i, x_j)$ is small
- otherwise, $d(x_i, x_j)$ is large.

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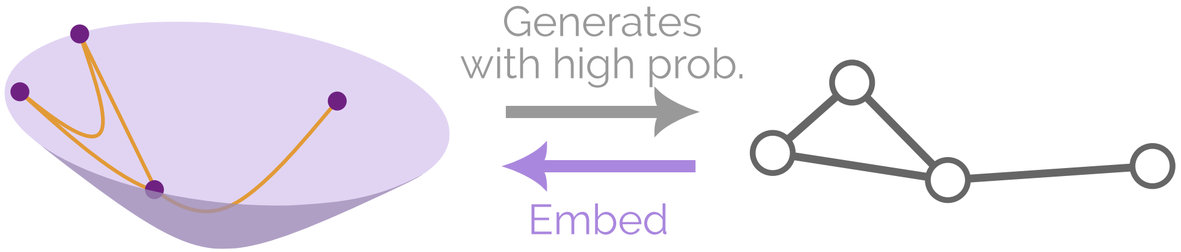
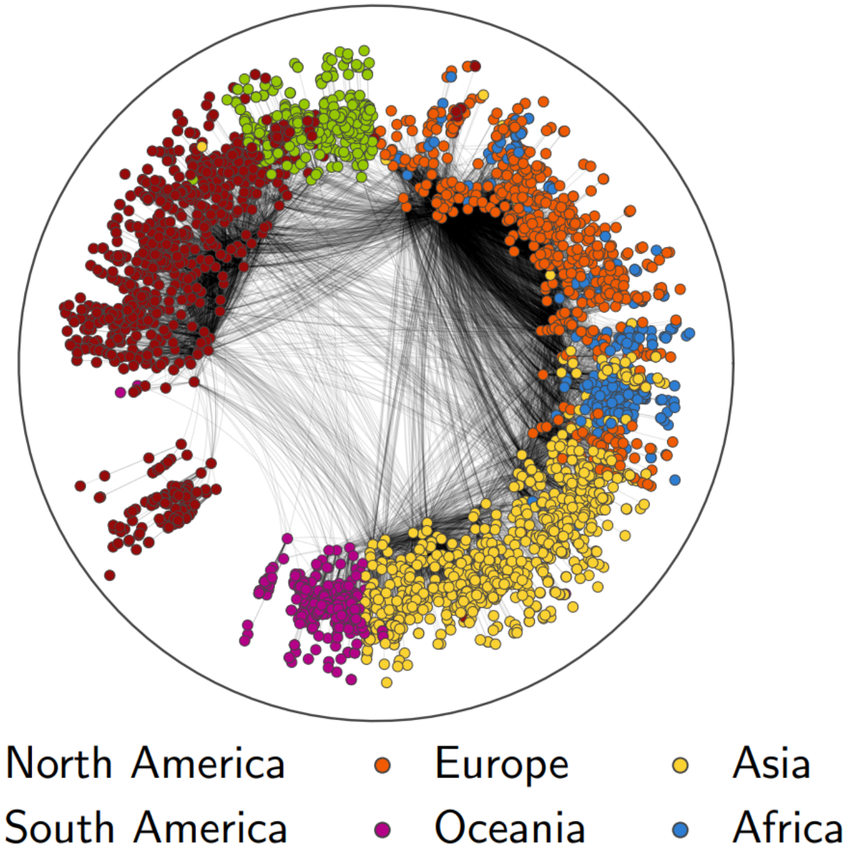
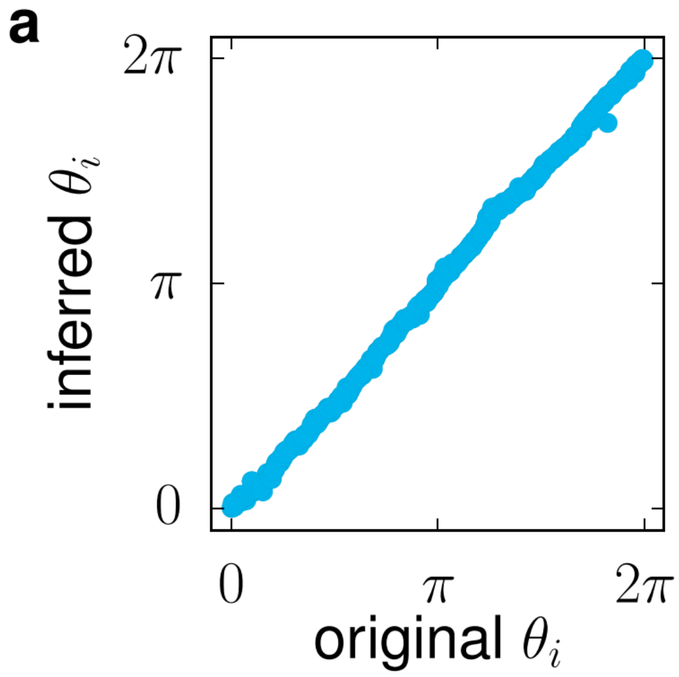
My project aims to find the **embeddings** which best **respect those constraints**.



Embedding of the world airport network[1]





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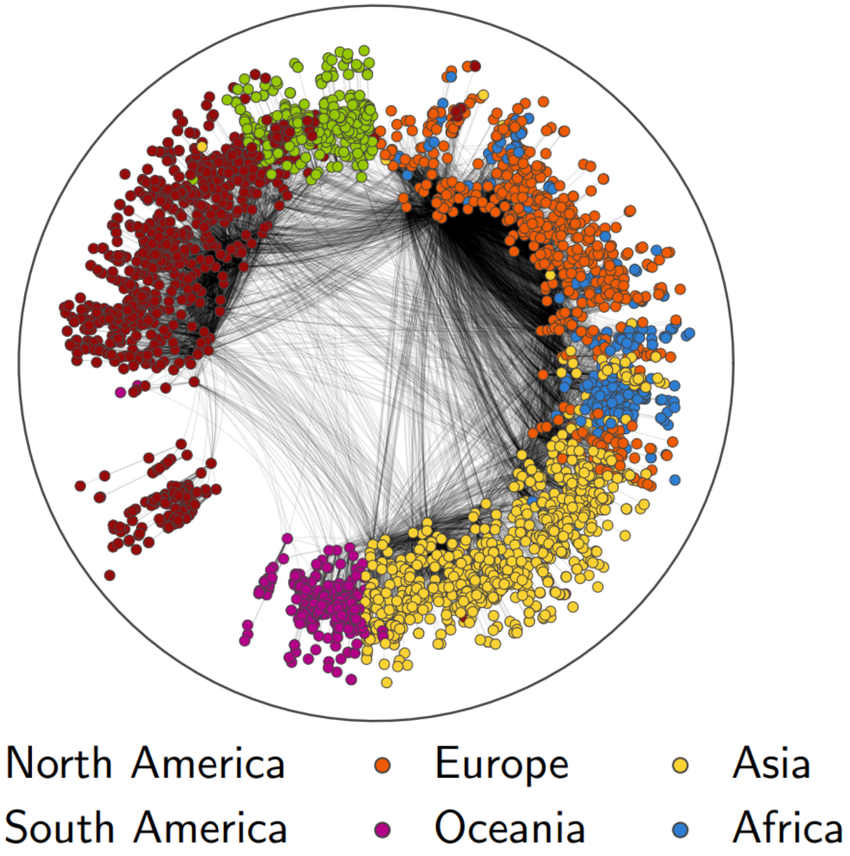
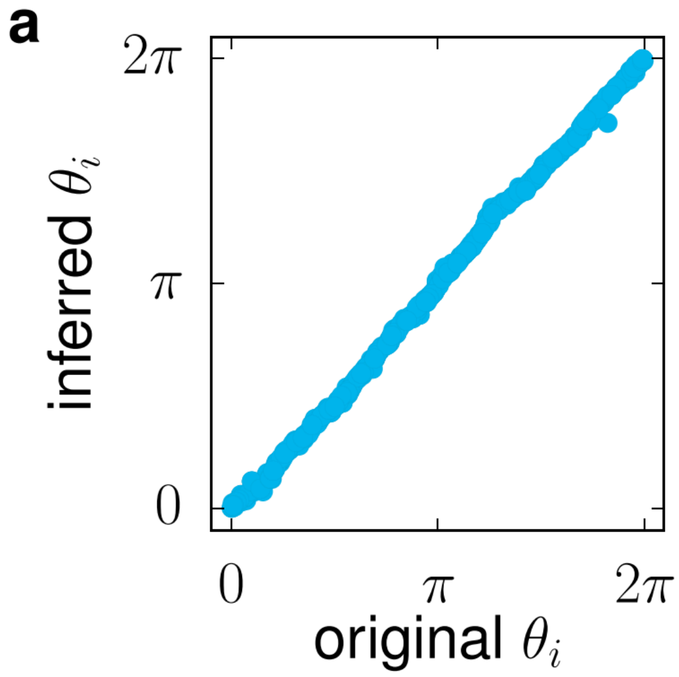
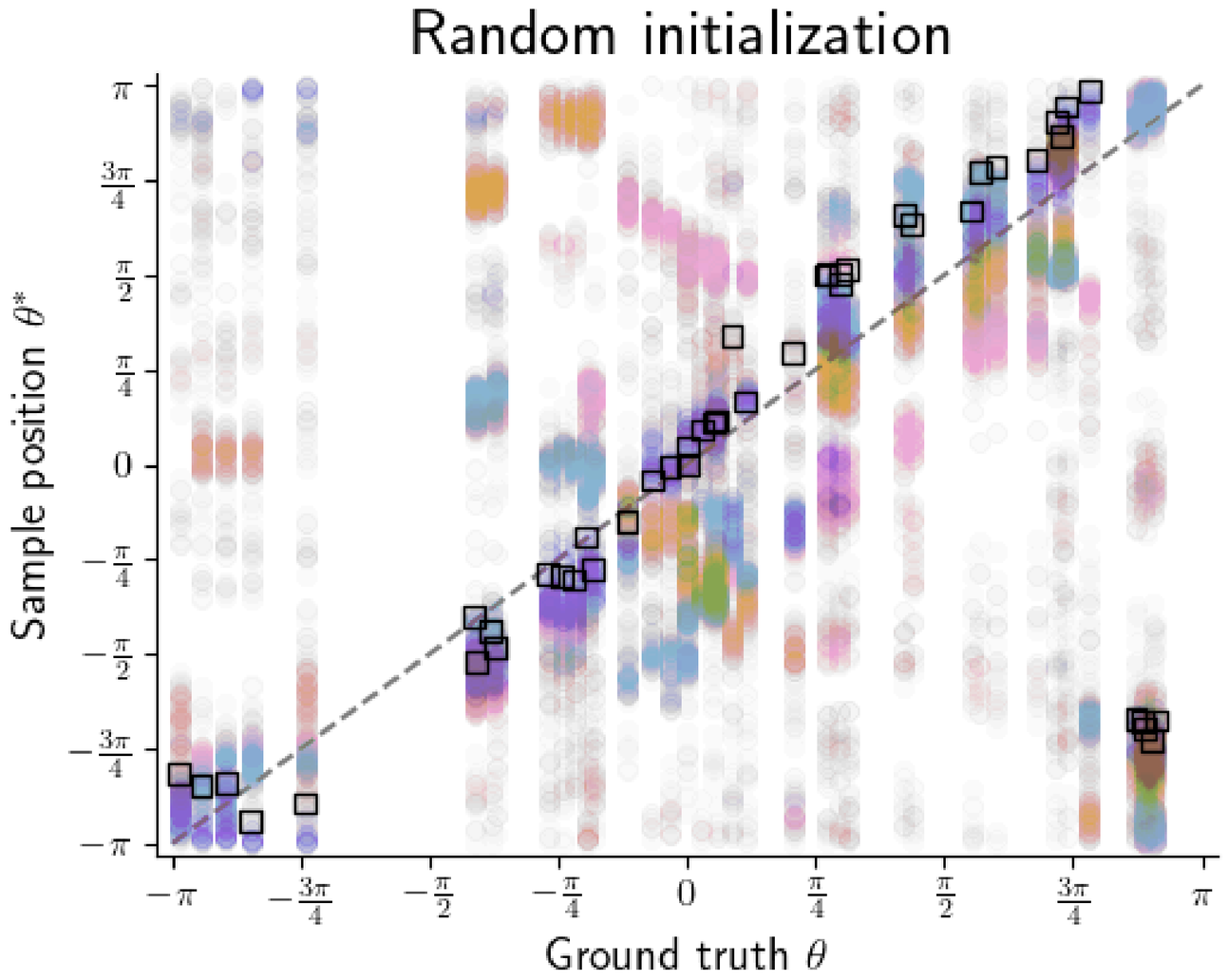




Current algorithms yield a **single embedding**. We ignore the margins of error and if there exists other good embeddings.

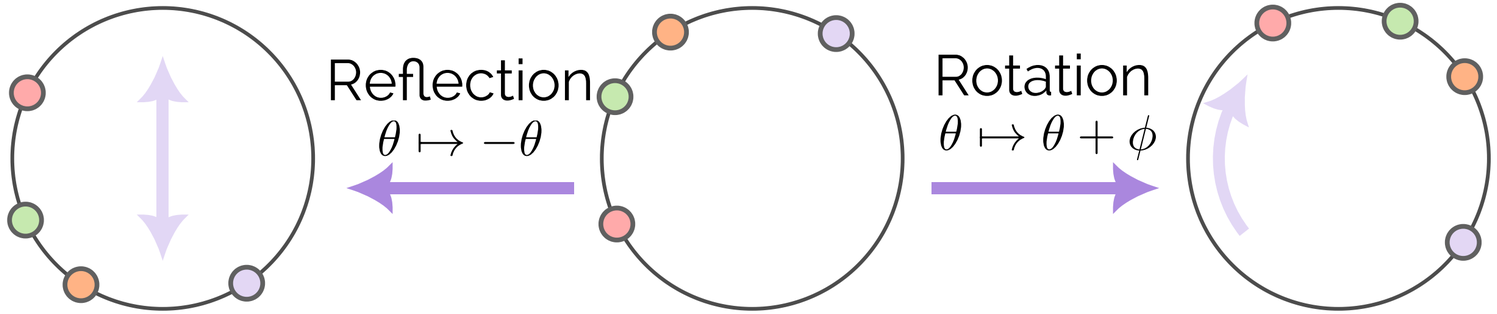
Preliminary results



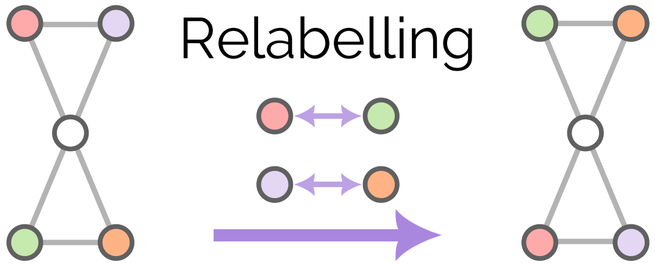
Some challenges of this optimization

There are many symmetries due to the graph and the metric space structure.

Space isometries

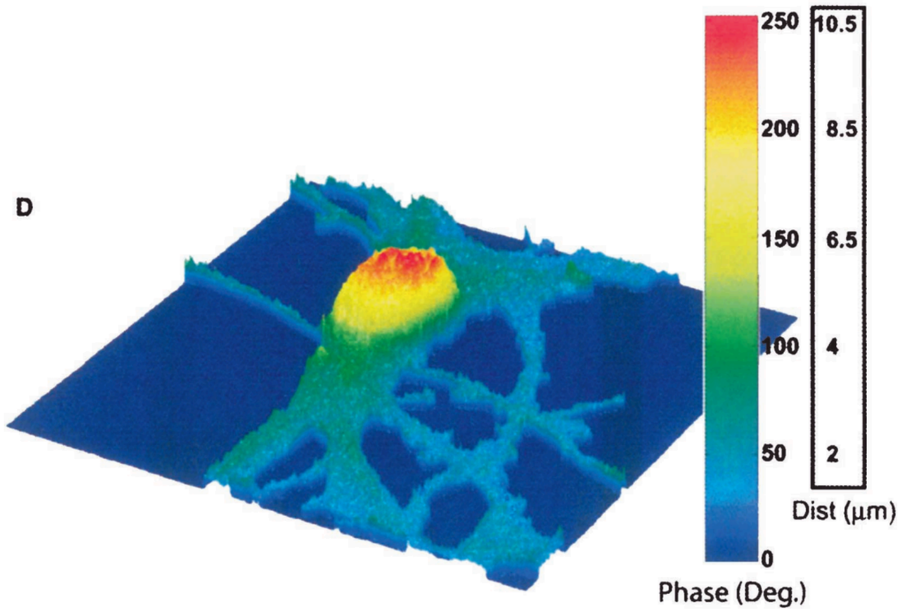


Graph automorphisms



Application to brain development

Pierre Marquet's team is imaging[1] neuron cultures with neurological disorders during their development.



We will analyse the **evolution of the embedding** of those brain networks in the hope of identifying a **geometric signature** for the neurological disorders.

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- The resilience of a complex system is related its **interaction structure**.
- The interaction structure can be represented as a **graph**.
- Geometric model: connected vertices are close and disconnected vertices are far.
- The **embedding** of the graph are the coordinates which best satisfy those conditions.
- Identify the **geometric signature** of neurological disorders in brain networks.