ALIFE 2020

LOCALIZATION, BISTABILITY AND OPTIMAL SEEDING OF CONTAGIONS ON HIGHER-ORDER NETWORKS

Guillaume St-Onge, Antoine Allard, Laurent Hébert-Dufresne

2020/07/15

Département de physique, de génie physique, et d’optique
Université Laval, Québec, Canada
Representations of complex systems

- **No structure**
  - Basic elements have **state**
  
- **Network structure**
  - Elements interact in **pairs**

- **Higher-order networks**
  - **Group** of elements interacting

- **State**: neuronal activity, political allegiance, species abundance
- **Pair** interaction: synapse, friendship, predator-prey relationship
- **Group** (higher-order) interaction: workplace environment, ecosystem
Contagion dynamics

No structure

Network structure

Higher-order networks

Susceptible  Infected

*Icons made by Freepik, catkuro, Smashicons and Pixel perfect from "www.flaticon.com"
Goal of the presentation

- Promote higher-order network (HON) representations of complex systems
- Introduce an accurate method to describe stochastic dynamics on HONs

Outline

1. Approximate master equations
2. Applications to contagion dynamics
   - Localization of epidemics
   - Bistability
   - Optimal seeding
Approximate master equations

Compartmental formalism

Transition rates

\[ \beta(\bullet, \bullet) \]

\[ \alpha(\bullet, \bullet) \]

\[ \text{cst.} \]
Mean-field equations for nodes

\[
\frac{ds_m}{dt} = 1 - s_m - m^r s_m.
\]

Approximate master equations for groups

\[
\frac{df_{n,i}}{dt} = (i + 1)f_{n,i+1} - if_{n,i} + (n - i) \left[ \beta(n,i) + \rho \right] f_{n,i} + (n - i + 1) \left[ \beta(n,i - 1) + \rho \right] f_{n,i - 1}.
\]

- \(s_m(t)\): fraction of susceptible nodes with membership \(m\)
- \(f_{n,i}(t)\): fraction of groups of size \(n\) with \(i\) infected
- \(\beta(n,i)\): local infection rate
- \(r(t), \rho(t)\): mean-field couplings
Epidemic localization

SIS model: $\beta(n, i) = \lambda i$

**Delocalized regime**

**Mesoscopic localization regime**
Localization regimes

Asymptotic analysis

\[ p_n \sim n^{-\gamma_n} \text{ with cut-off } n_{\text{max}} \]

Membership distribution \[ g_m \sim m^{-\gamma_m} \text{ with cut-off } m_{\text{max}} = n_{\text{max}} \]
Bistability emerges from nonlinear interactions

Simple model of social contagion

\[ \beta(n, i) = \lambda i^\nu \]

- \( \nu < 1 \): inhibition effect
- \( \nu = 1 \): SIS model
- \( \nu > 1 \): reinforcement effect
Influence maximization

**Goal:** *Maximize $\dot{I}(0)$ by distributing wisely $I(0) = \epsilon \ll 1$.  

**Rules**

- We set $\lambda > \lambda_c$ so that $I^* = 0$ is unstable.
- You can choose among two approaches:
  1. *Influential spreaders:* engineer node set $\{s_m(0)\}$
  2. *Influential groups:* engineer group set $\{f_{n,i}(0)\}$
- The unchosen set is distributed randomly, i.e.

$$f_{n,i}(0) = \binom{n}{i} \epsilon^i (1 - \epsilon)^{n-i} \quad \text{or} \quad s_m = 1 - \epsilon \ \forall m.$$
Influential spreaders

Infect nodes with highest available membership $m$

Influential groups

Optimal strategy

Favor most profitable group configurations $(n, i)$ as measured from
$R(n, i) = \beta(n, i)(n - i)/i$
Influential groups beat influential spreaders in nonlinear contagions
What can higher-order network representations do for you?

- New insights due to the focus on groups of elements
- Analytical results to guide further exploration
- The framework presented can be applied to various dynamical processes
  - Voter models, evolutionary game theory, etc.

\[
\frac{df_{n,i}}{dt} = (i + 1) \left[ \alpha(n, i + 1) + \rho_1 \right] f_{n,i+1} - i \left[ \alpha(n, i) + \rho_1 \right] f_{n,i} - (n - i) \left[ \beta(n, i) + \rho_2 \right] f_{n,i} + (n - i + 1) \left[ \beta(n, i - 1) + \rho_2 \right] f_{n,i-1}.
\]
Aknowledgments

Epidemic localization
Vincent Thibeault, Antoine Allard, Louis J. Dubé, Laurent Hébert-Dufresne

Bistability and optimal seeding
Iacopo Iacopini, Giovanni Petri, Alain Barrat, Vito Latora, Laurent Hébert-Dufresne

Funding and computational ressources