

# The hidden low-dimensional dynamics of large neuronal networks

Frontiers in Neurophotonics 2022

Patrick Desrosiers



**Dynamica**

**CIMMUL** Centre Interdisciplinaire en Modélisation  
Mathématique de l'Université Laval



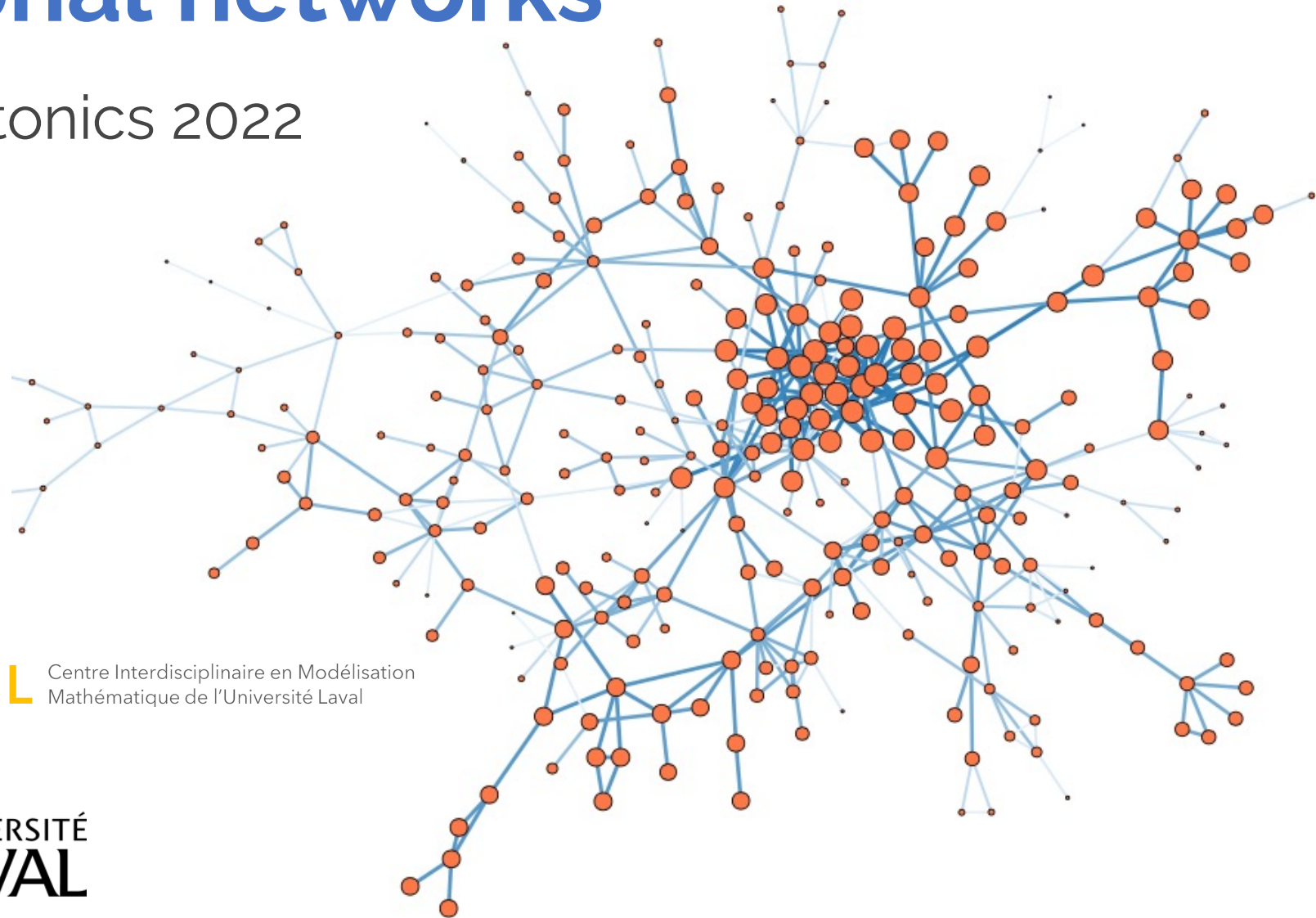
CENTRE DE RECHERCHE

**CERVO**

BRAIN RESEARCH CENTRE



UNIVERSITÉ  
**LAVAL**

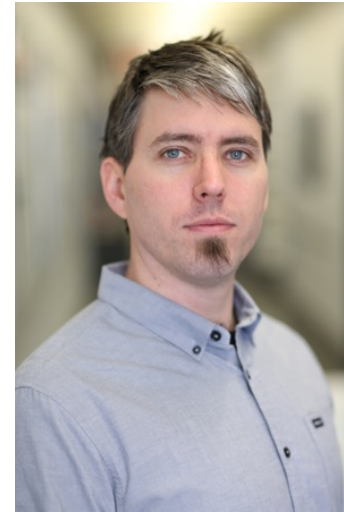




Daniel Côté



Paul De Koninck



Benoit Labonté



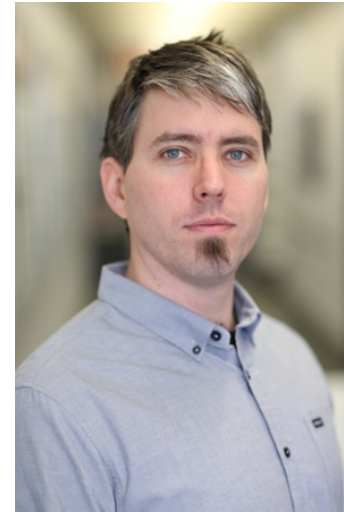
Pierre Marquet



Daniel Côté



Paul De Koninck



Benoit Labonté



Pierre Marquet



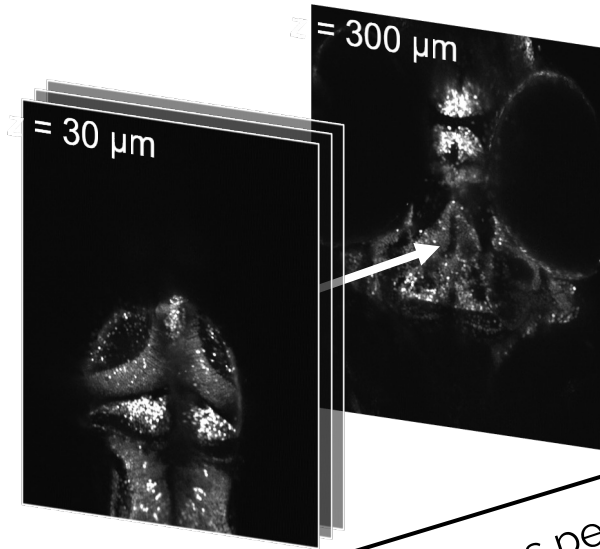


Antoine Légaré

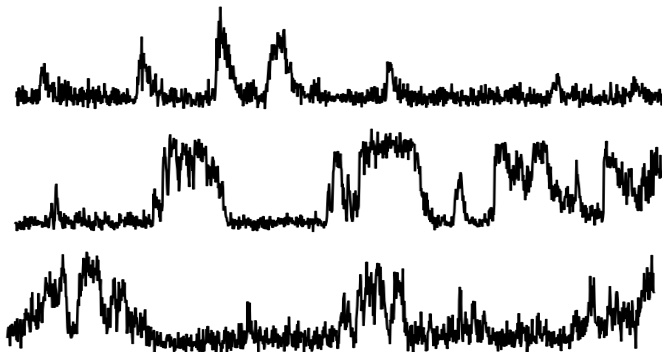
# Neuronal activity in zebrafish brain



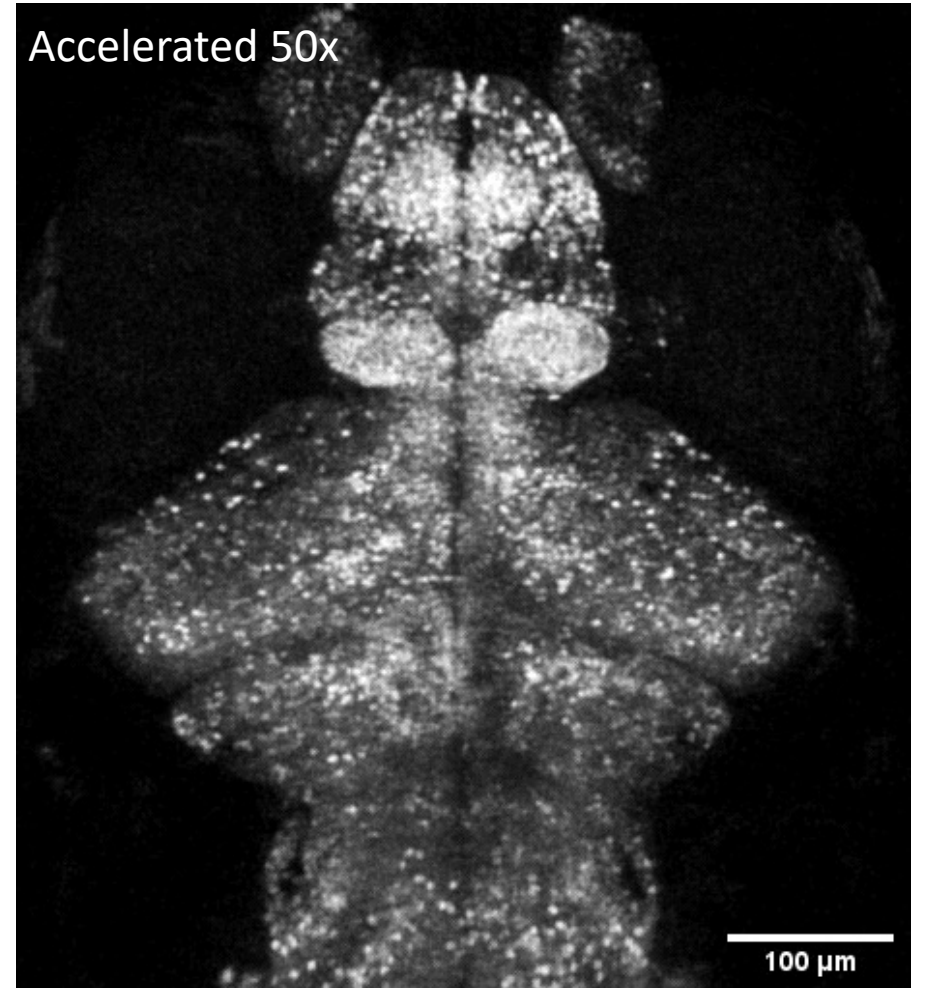
Larval zebrafish



29 planes per second



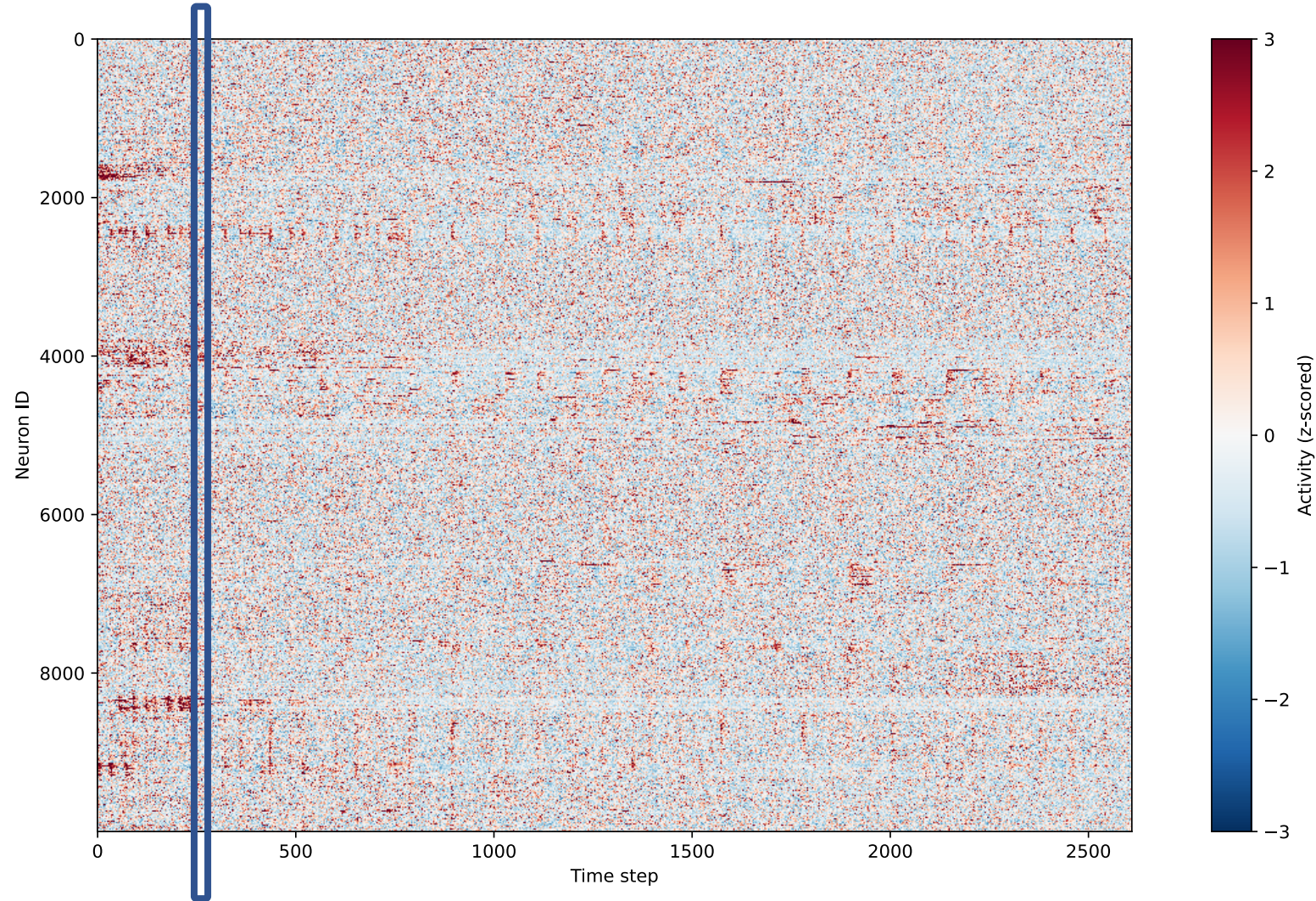
61500 neurons on average



Source: Paul De Koninck' Lab at CERVO



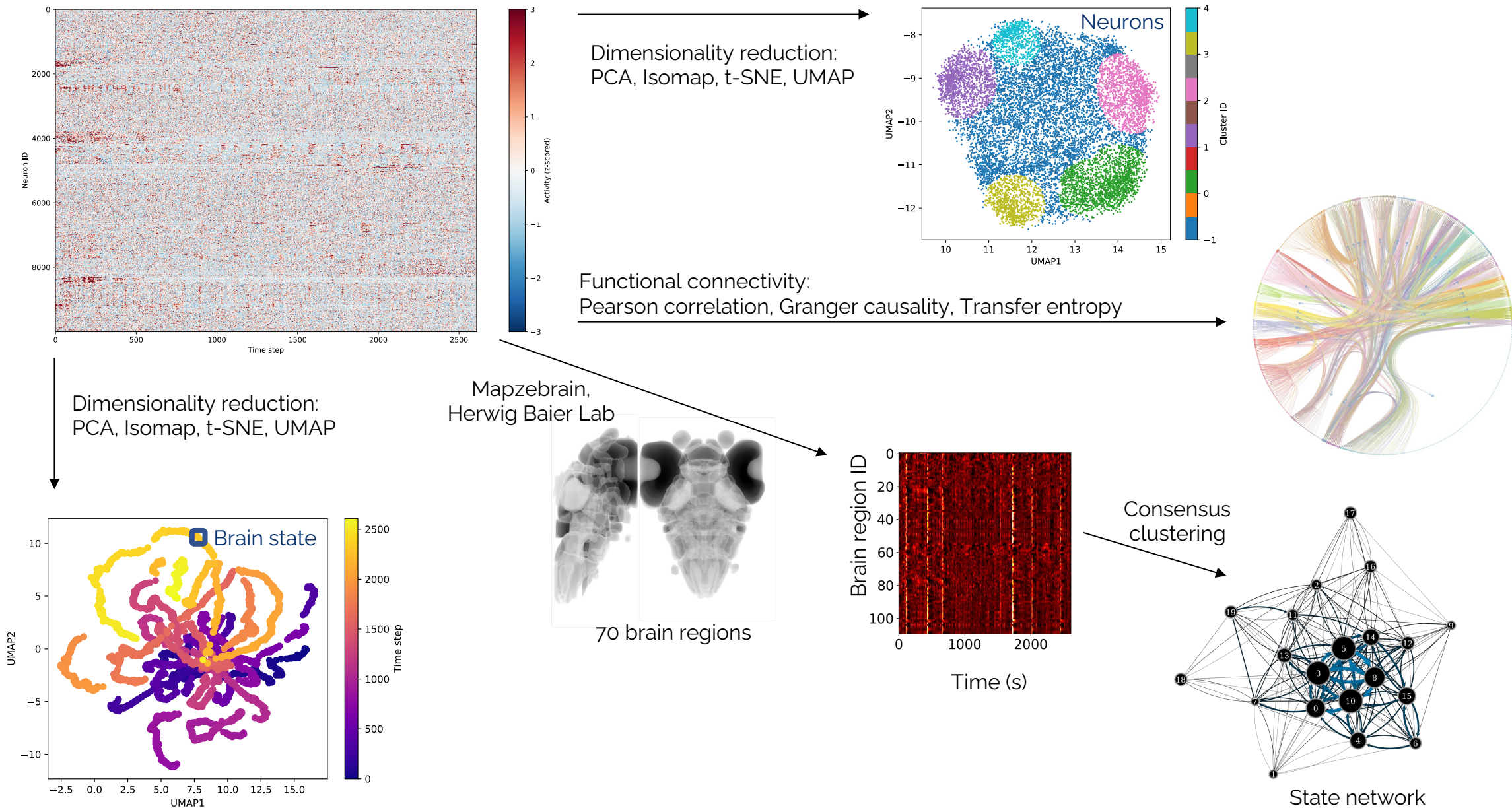
# Neuronal activity in zebrafish brain: data



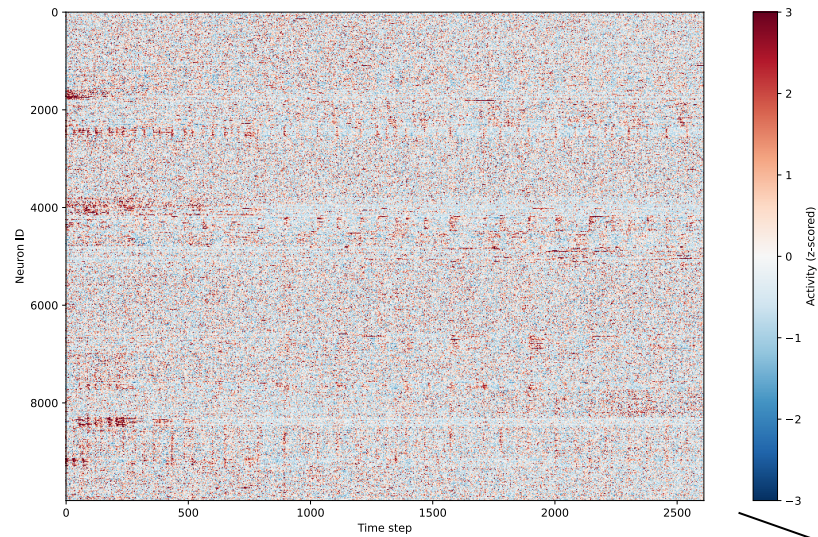
Brain state  
at time step  $t$



# Neuronal activity in zebrafish brain: possible analyses



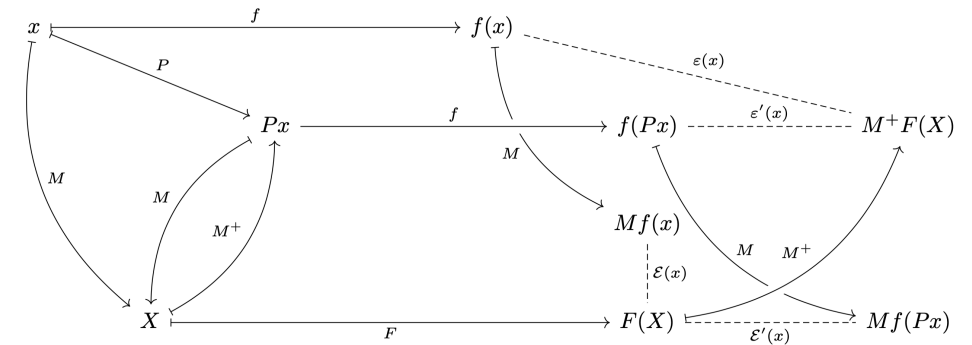
# Neuronal activity in zebrafish brain: possible analyses



Mathematical modeling of the whole dynamics ?

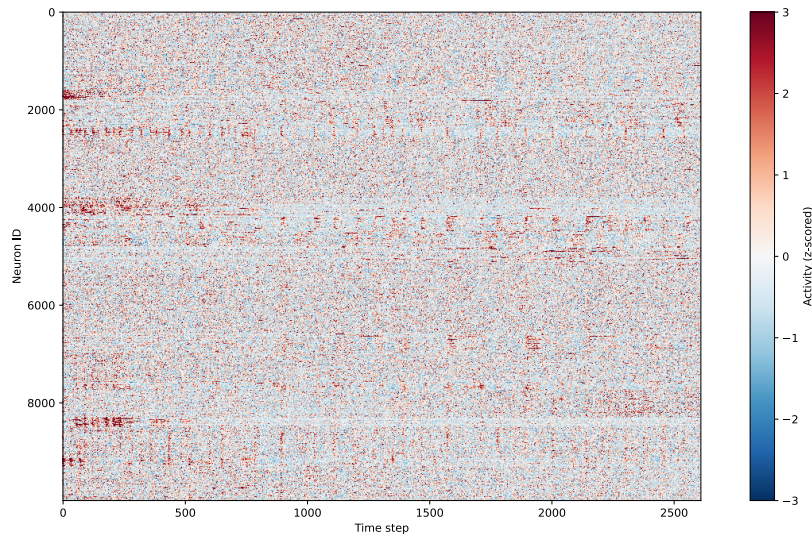
$$\dot{X} = \frac{d(R \circ x)}{dt} = \mathcal{U}[R] \circ x = J_R \circ f \circ x = F \circ R \circ x = F \circ X,$$

$$\dot{X} = \mathcal{U}[R] \circ x = J_R \circ f \circ x.$$

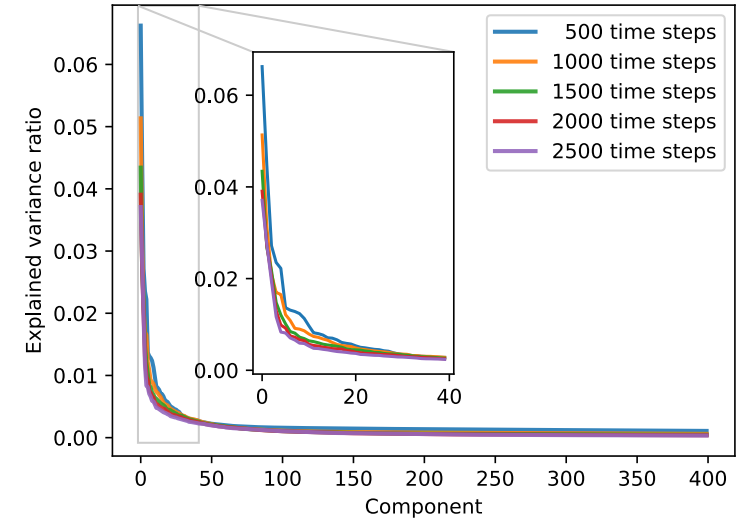
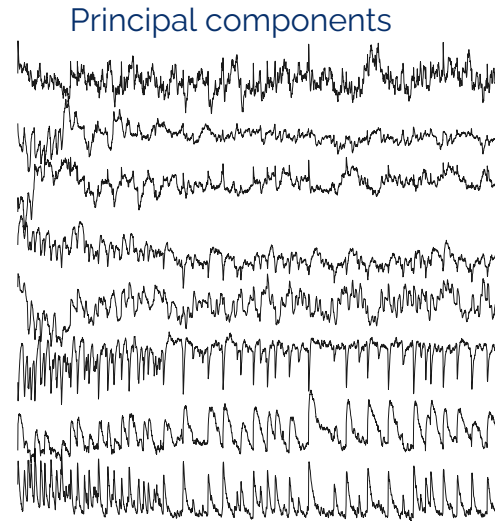




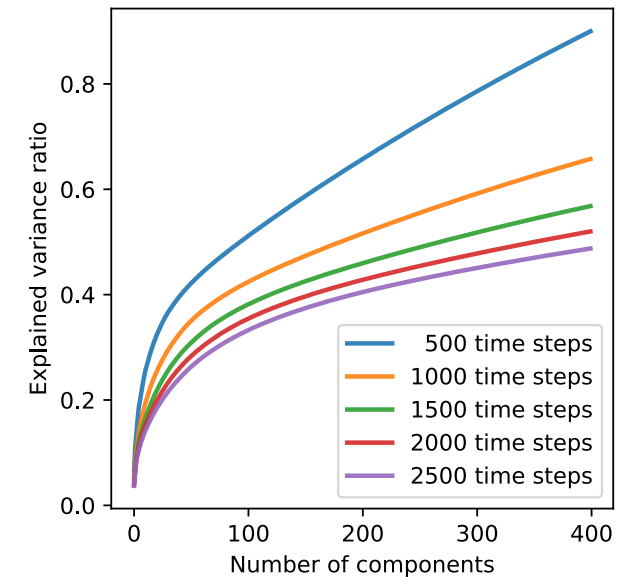
# PCA and low-dimensionality



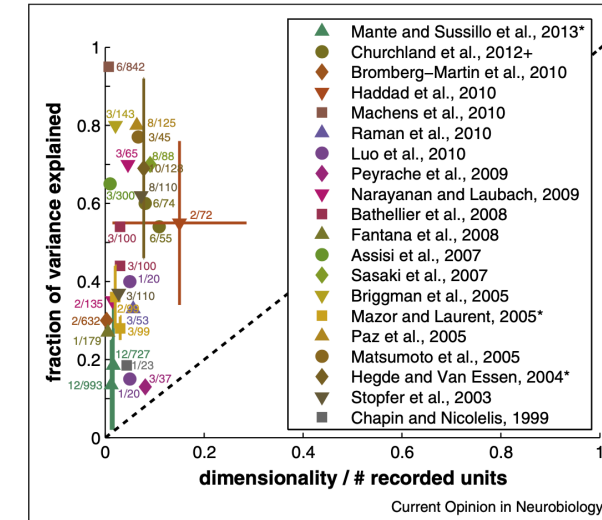
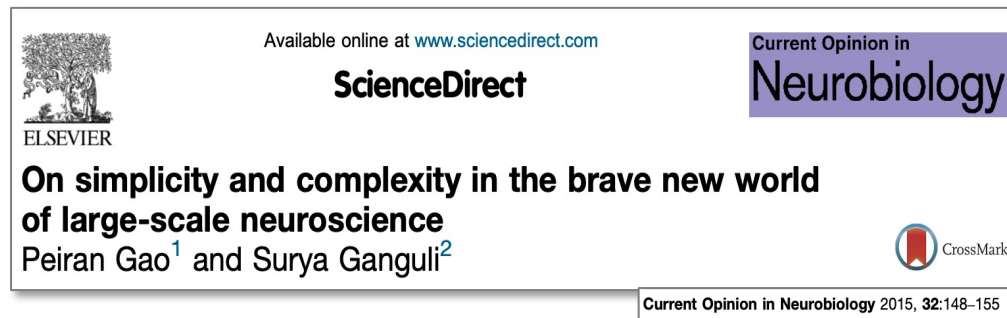
PCA →



- Each neuron is approximately equal to a linear combination of a few principal components.
- Suggests that the dimensionality of the whole brain is much smaller than the number of neurons.



# PCA and low-dimensionality

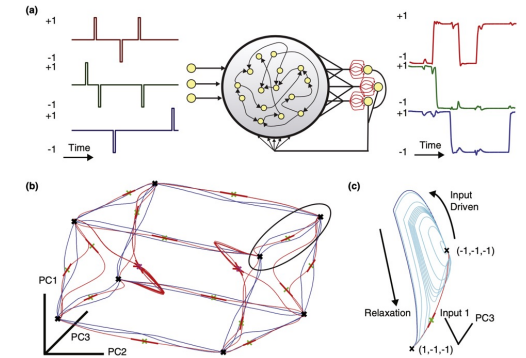
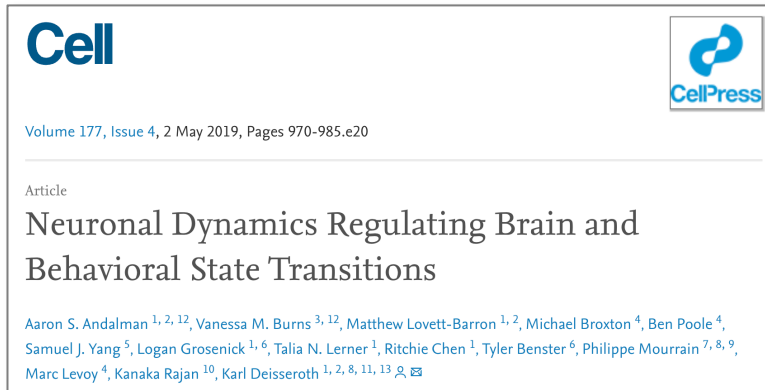


In many experiments (e.g. in insect [20,23–26] olfactory systems, mammalian olfactory [26,27], prefrontal [21,22\*,28–30], motor and premotor, [31,32], somatosensory [33], visual [34,35], hippocampal [36], and brain stem [37] systems) a *much* smaller number of dimensions than the number of recorded neurons captures a large amount of variance in neural firing rates.

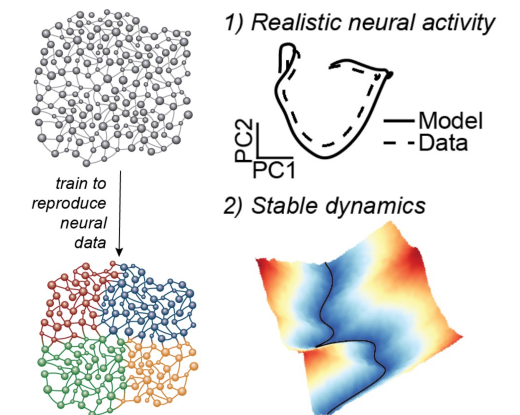
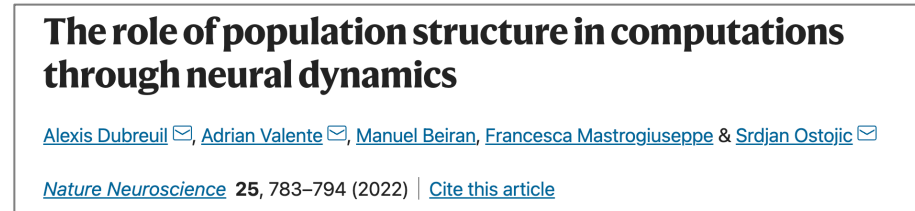
**Why should we expect low dimensionality for large neuronal networks ?**



# Neuronal activity: resurgence of the dynamical system approach



D. Sussillo 2014



M. G. Perich et al. 2020

# Firing rate model for recurrent neural networks

Grossberg, Amari, Wilson–Cowan, Hopfield, ...

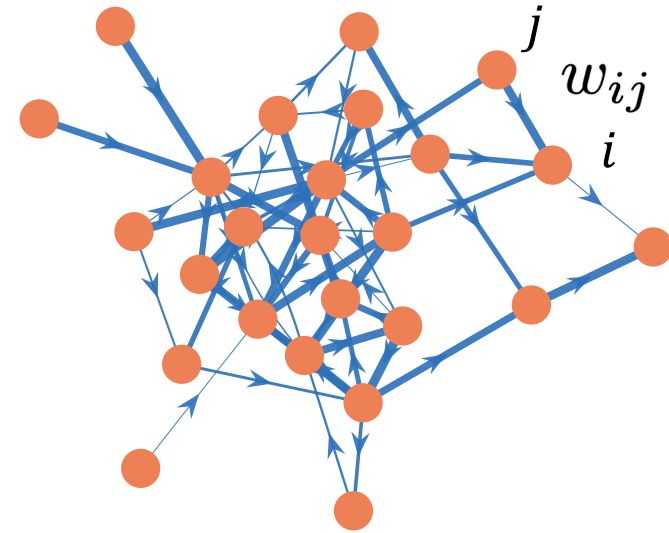
$$\frac{dx_i}{dt} = -x_i + \sigma\left(\sum_j w_{ij}x_j - \mu_i\right)$$

$x_i(t)$  = activity of neuron  $i$  at time  $t$

$\mu_i$  = activation threshold of neuron  $i$

$w_{ij}$  = weight of the connection from neuron  $j$  to neuron  $i$

$$i, j \in \{1, 2, \dots, N\}$$



## NEURAL NETWORK DYNAMICS

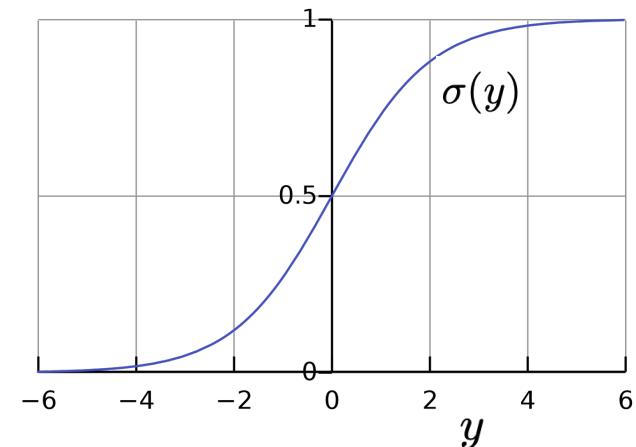
Annual Review of Neuroscience

Vol. 28:357-376 (Volume publication date 21 July 2005)

First published online as a Review in Advance on March 22, 2005

<https://doi.org/10.1146/annurev.neuro.28.061604.135637>

Tim P. Vogels, Kanaka Rajan, and L.F. Abbott



# Firing rate model for recurrent neural networks

Grossberg, Amari, Wilson–Cowan, Hopfield, ...

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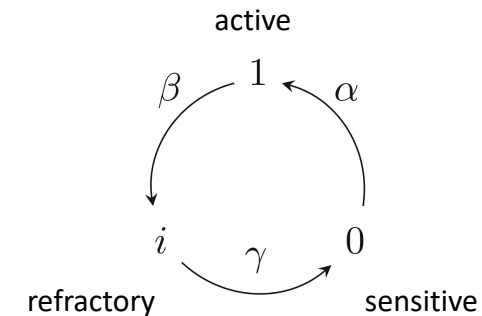


Original Article | [Open Access](#) | [Published: 05 September 2022](#)

## Beyond Wilson–Cowan dynamics: oscillations and chaos without inhibition

[Vincent Painchaud](#), [Nicolas Doyon](#) & [Patrick Desrosiers](#)

[Biological Cybernetics](#) (2022) | [Cite this article](#)





# Firing rate model for recurrent neural networks

Grossberg, Amari, Wilson–Cowan, Hopfield, ...

$$\frac{dx}{dt} = -x + \sigma(Wx - \mu)$$

$x = N \times 1$  network state vector at time  $t$

$\mu = N \times 1$  vector of thresholds

$W = N \times N$  weight matrix

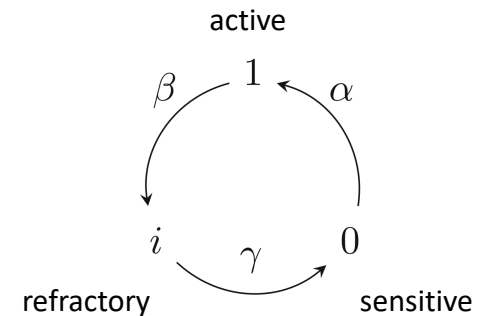


Original Article | [Open Access](#) | [Published: 05 September 2022](#)

## Beyond Wilson–Cowan dynamics: oscillations and chaos without inhibition

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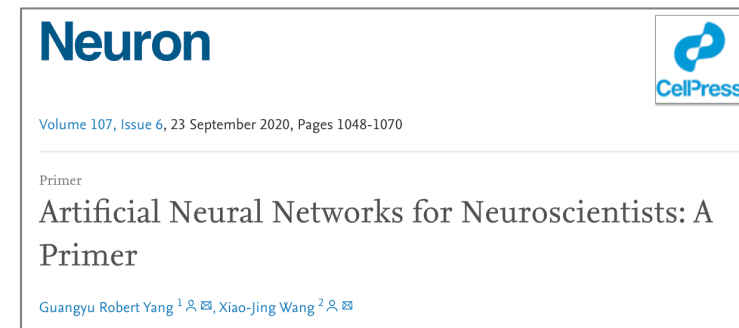
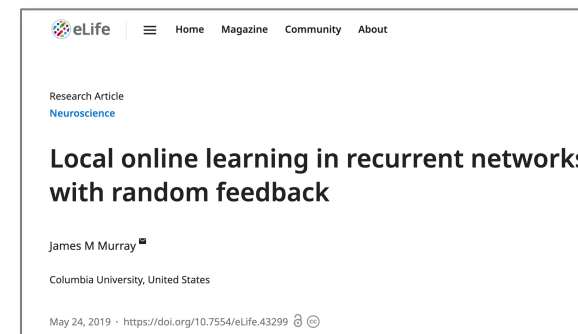
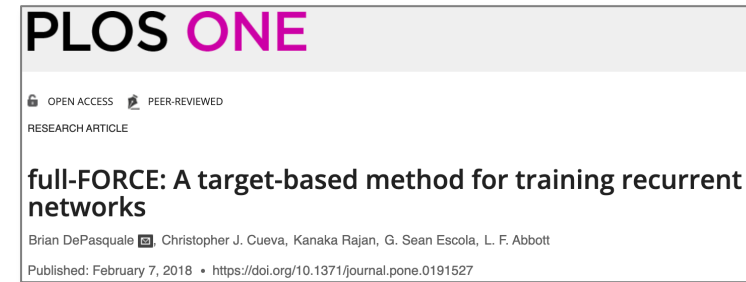
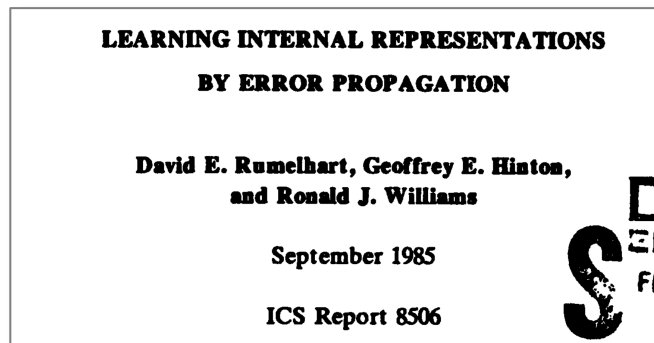
[Biological Cybernetics](#) (2022) | [Cite this article](#)



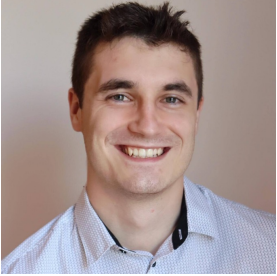
# Recurrent neural networks can be trained to fit the data

Unknown parameters

$$\frac{dx}{dt} = -x + \sigma(Wx - \mu)$$



# Recurrent neural networks can be trained to fit the data



Jérémie Gince



Anthony Drouin



Simon Hardy



Daniel Côté



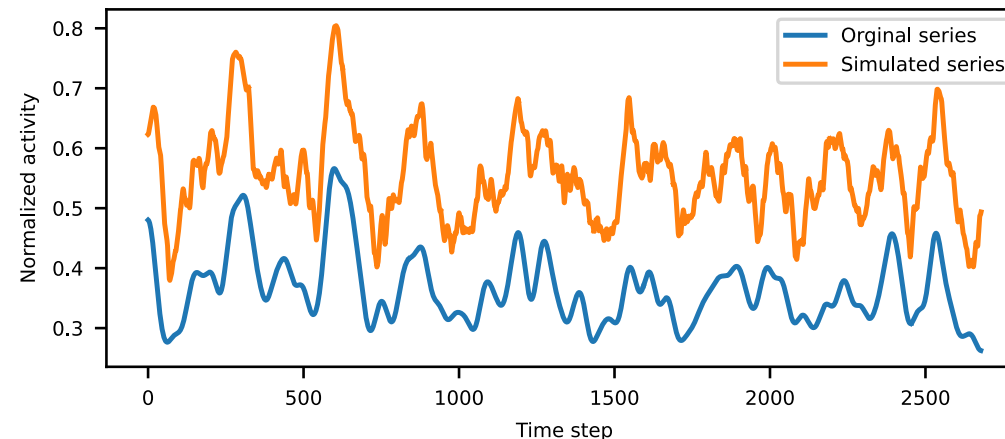
<https://github.com/NeuroTorch/NeuroTorch>

**Current Version (v0.0.1-alpha)**

- Image classification with spiking networks.
- Classification of spiking time series with spiking networks.
- Time series classification with spiking or Wilson-Cowan.
- Reconstruction/Prediction of time series with Wilson-Cowan.
- Reconstruction/Prediction of continuous time series with spiking networks.
- Backpropagation Through Time.



Antoine Légaré



Worst solution  
RMSE=0.19



# Recurrent neural networks can be trained to fit the data



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- Reconstruction/Prediction of continuous time series with spiking networks.
- Backpropagation Through Time.

After training:

$$\frac{dx}{dt} = -x + \sigma(\boxed{W}x - \boxed{\mu})$$

Learned parameters

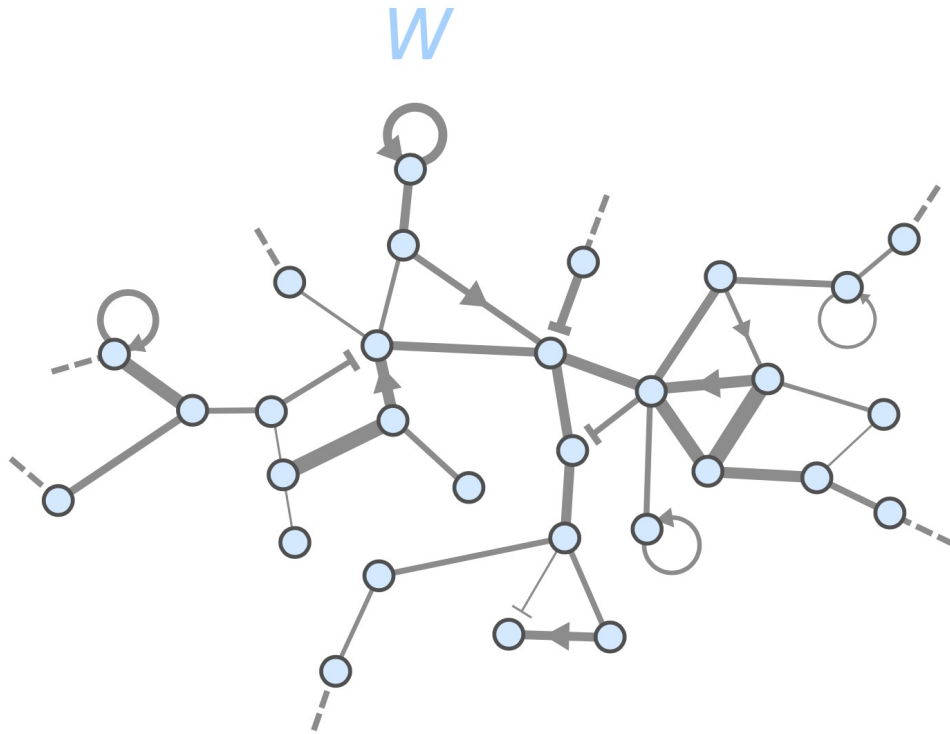
High-dimensional  
dynamical system

**How to reduce the dimensionality of large dynamical systems?**

**Is it justified to reduce the dimensionality?**

# Our point of view: complex systems theory

Complex network

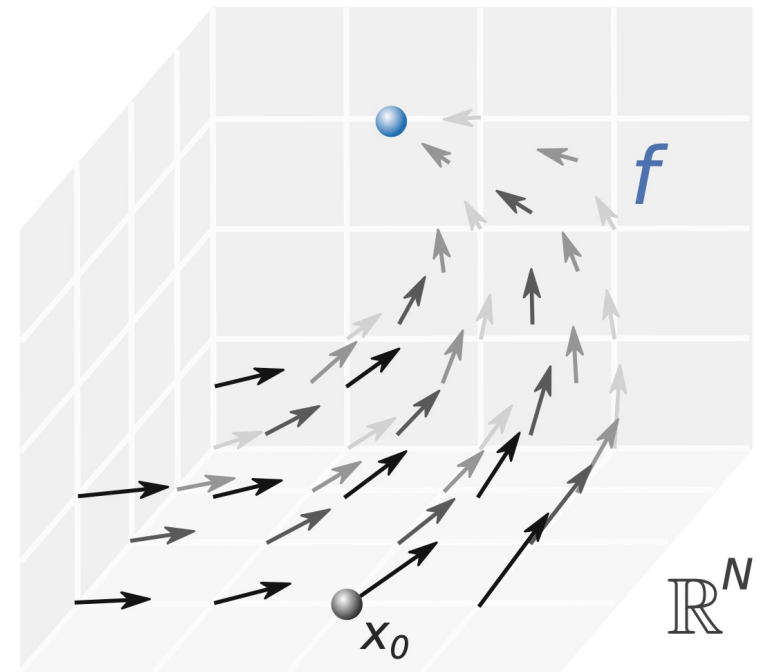


Vector field



High-dimensional dynamics

$$\dot{x} = f(x; W)$$

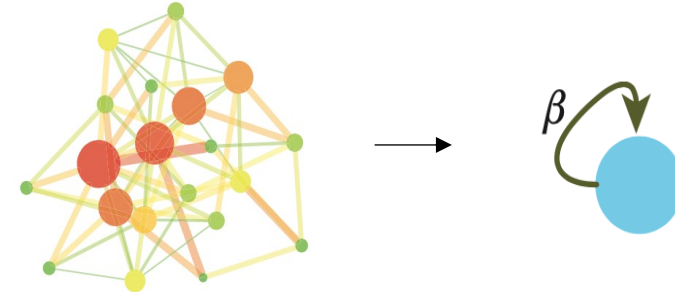


# Our first inspiration: Dimension reduction to study resilience

## Universal resilience patterns in complex networks

Jianxi Gao<sup>1\*</sup>, Baruch Barzel<sup>2\*</sup> & Albert-László Barabási<sup>1,3,4,5</sup>

18 FEBRUARY 2016 | VOL 530 | NATURE | 307



Dimension reduction based on degrees:

$$\frac{dx_i}{dt} = F(x_i) + \sum_j W_{ij} G(x_i, x_j) \quad \longrightarrow \quad \frac{dx}{dt} = F(x) + \beta G(x, x)$$

where

$$x = \frac{\sum_{i,j} W_{ij} x_j}{\sum_{i,j} W_{ij}} = \text{degree-weighted average activity}$$

$$\beta = \frac{\sum_{i,j,k} W_{ij} W_{jk}}{\sum_{i,j} W_{ij}} = \text{degree-weighted average degree}$$

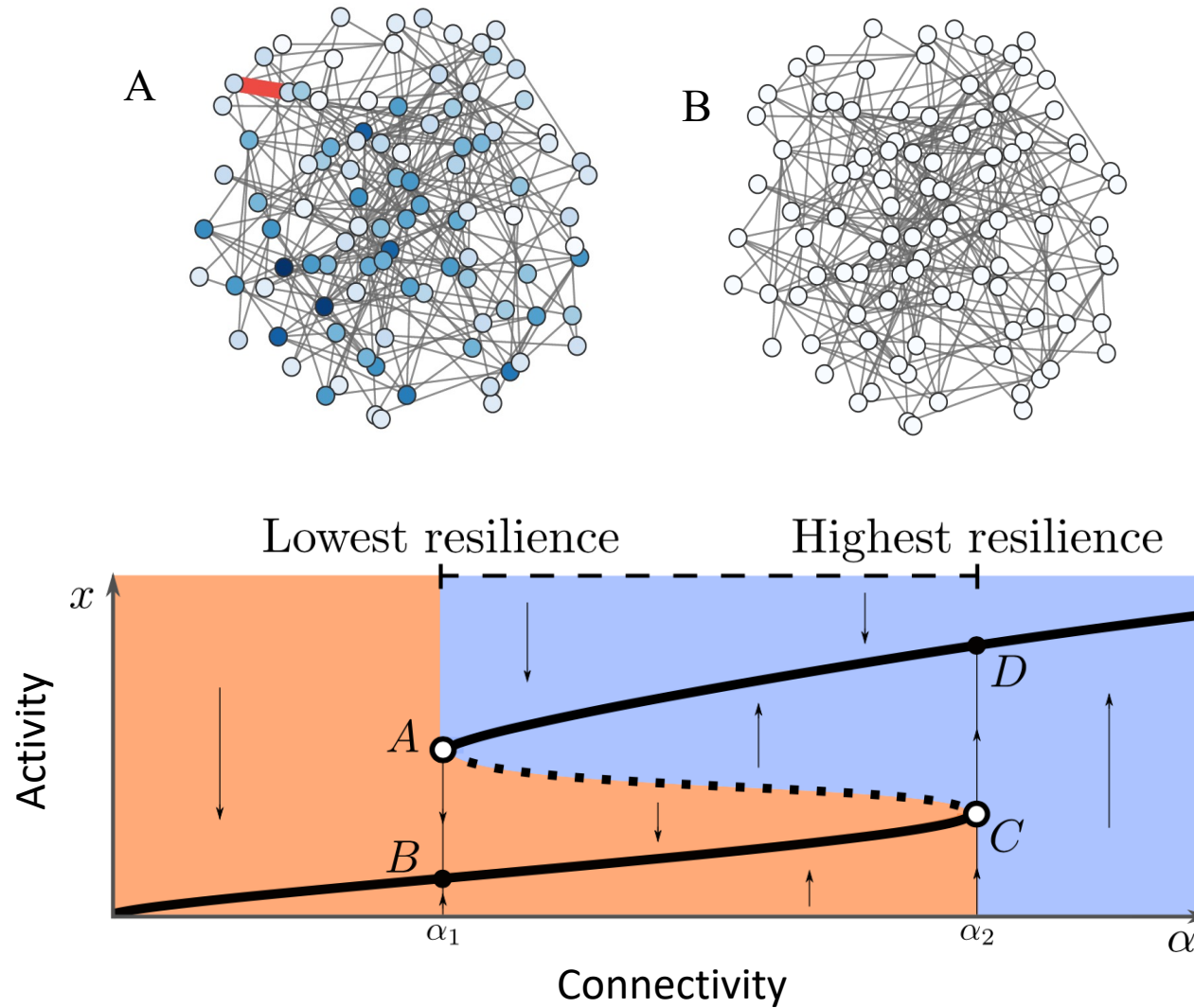
$$i, j, k \in \{1, \dots, N\}$$

**Success:** The reduction allows studying resilience.

**Problem:** The reduction does not work well with all networks.



# Resilience in complex systems

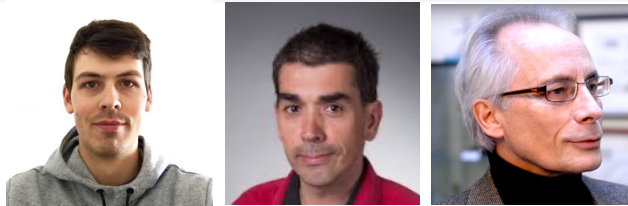


# More complete solutions:

PHYSICAL REVIEW X 9, 011042 (2019)

## Spectral Dimension Reduction of Complex Dynamical Networks

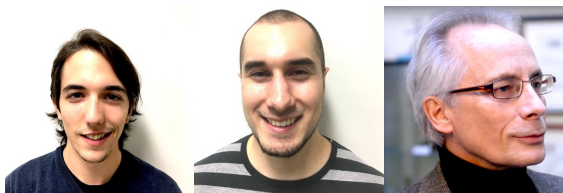
Edward Laurence,<sup>1,2</sup> Nicolas Doyon,<sup>2,3,4</sup> Louis J. Dubé,<sup>1,2</sup> and Patrick Desrosiers<sup>1,2,4</sup>



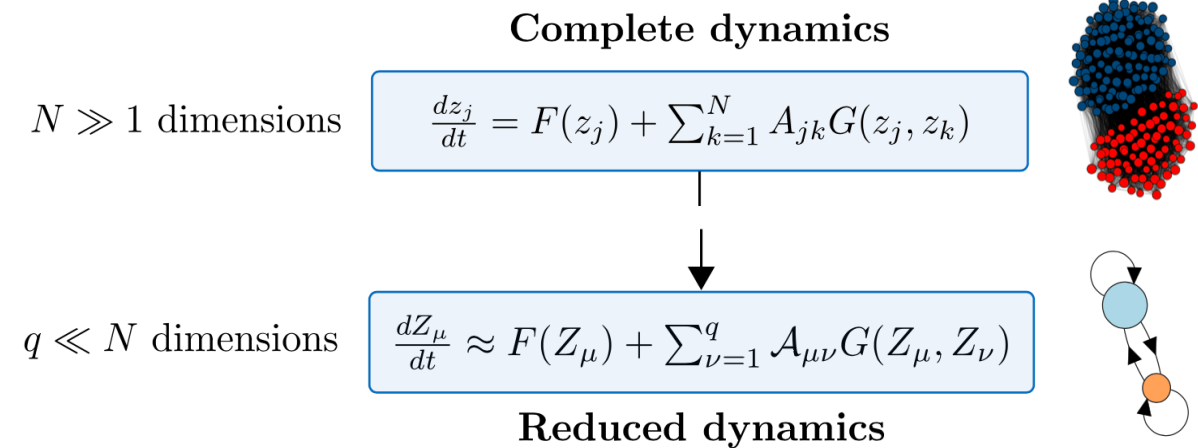
PHYSICAL REVIEW RESEARCH 2, 043215 (2020)

## Threefold way to the dimension reduction of dynamics on networks: An application to synchronization

Vincent Thibeault<sup>1,2,\*</sup> Guillaume St-Onge<sup>1,2</sup> Louis J. Dubé,<sup>1,2</sup> and Patrick Desrosiers<sup>1,2,3,†</sup>



## DART: Dynamics Approximate Reduction Technique



PAPER

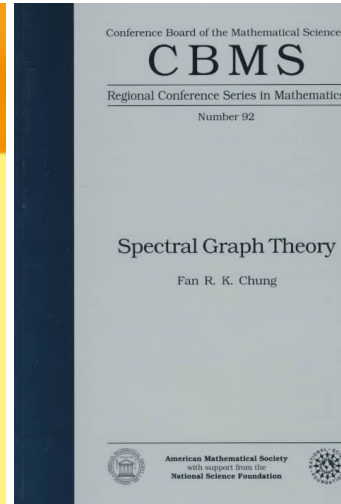
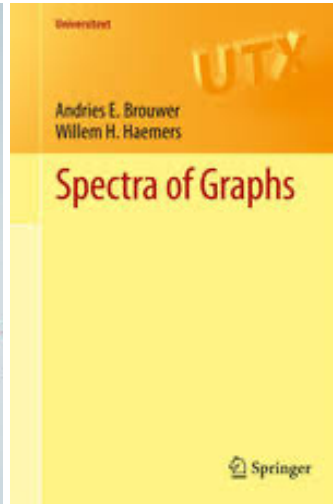
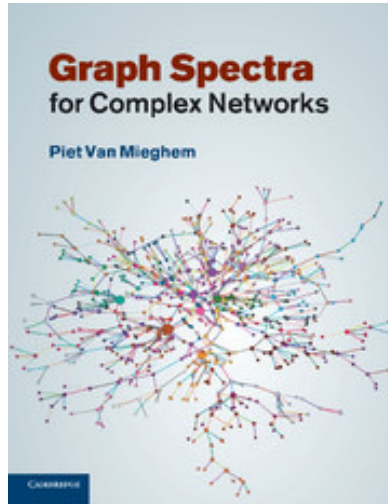
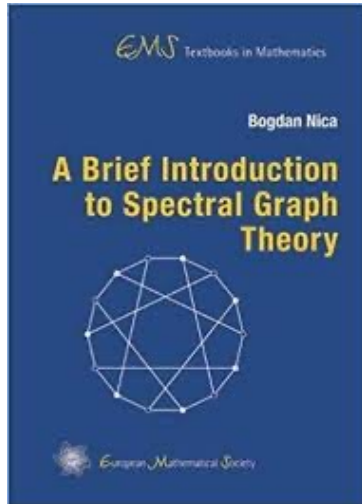
## Dimension reduction of dynamics on modular and heterogeneous directed networks

Marina Vegué,<sup>a,b,\*</sup> Vincent Thibeault,<sup>a,b</sup> Patrick Desrosiers<sup>a,b,c</sup> and Antoine Allard<sup>a,b</sup>



# Theoretical framework: Spectral Graph Theory and Matrix factorization

Good sources of information:



## Learning the parts of objects by non-negative matrix factorization

Daniel D. Lee\* & H. Sebastian Seung\*†

NATURE | VOL 401 | 21 OCTOBER 1999 | [www.nature.com](http://www.nature.com)

IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE, VOL. 32, NO. 1, JANUARY 2010

## Convex and Semi-Nonnegative Matrix Factorizations

Chris Ding, *Member, IEEE*, Tao Li, and Michael I. Jordan, *Fellow, IEEE*

Old treasure:

## A GENERALIZED INVERSE FOR MATRICES

By R. PENROSE

Communicated by J. A. TODD

*Received 26 July 1954*

Recent treasure:

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 60, NO. 8, AUGUST 2014

## The Optimal Hard Threshold for Singular Values is $4/\sqrt{3}$

Matan Gavish, *Student Member, IEEE*, and David L. Donoho, *Member, IEEE*

# Our latest approach: Singular Value Decomposition

arXiv:2208.04848

The low-rank hypothesis of complex systems:  
From empirical and theoretical evidence to the emergence of higher-order interactions

Vincent Thibeault,<sup>1,2,\*</sup> Antoine Allard,<sup>1,2,†</sup> and Patrick Desrosiers<sup>1,2,3,‡</sup>

$$W = U \Sigma V^T$$

*Orthogonal  $N \times N$  matrix*      *Diagonal  $N \times N$  matrix*      *Orthogonal  $N \times N$  matrix*

Rank : number of linearly independent rows/columns of a matrix



$$\begin{array}{c}
 W \approx U_n \Sigma_n V_n^T \\
 \begin{array}{ccc}
 & \begin{array}{c} \sigma_1 \\ \vdots \\ \sigma_n \end{array} & \\
 & n \times n & \\
 \begin{array}{c} N \times n \end{array} & & \begin{array}{c} n \times N \end{array}
 \end{array}
 \end{array}$$

Optimal low-rank approximation  
 ( Eckart-Young theorem )  
 ( Exact for  $n = r$  )

rank 1

$W = \sigma_1 X_1 + \sigma_2 X_2 + \dots + \sigma_r X_r$

largest contribution                      smallest contribution

# Flags have low rank

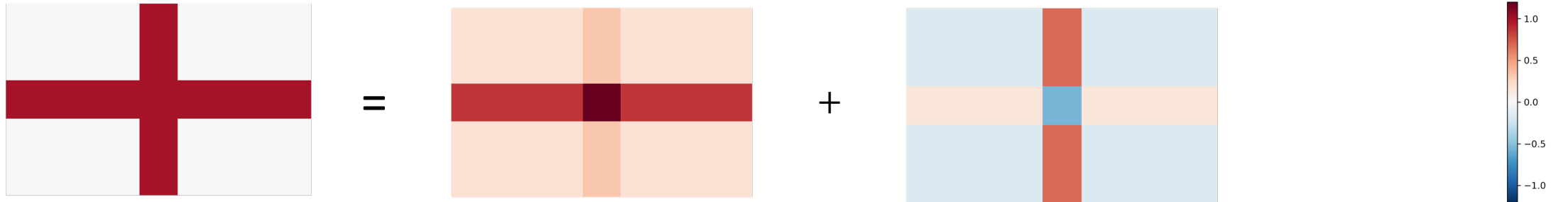


Inspired by a lecture by Alex Townsend on [Rapidly Decreasing Singular Values](#)

# France's flag has rank 1

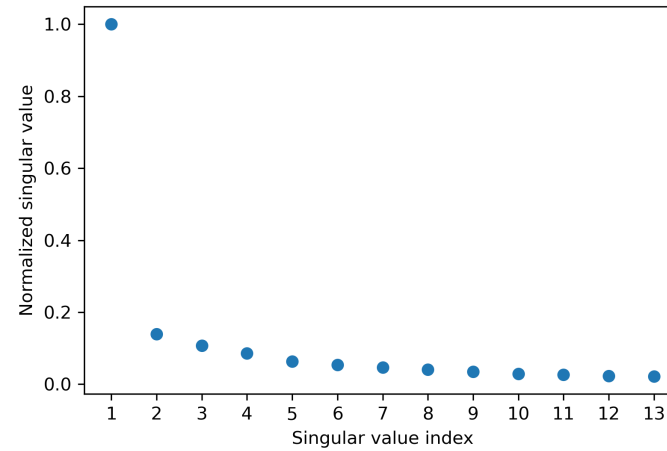


# England's flag has rank 2



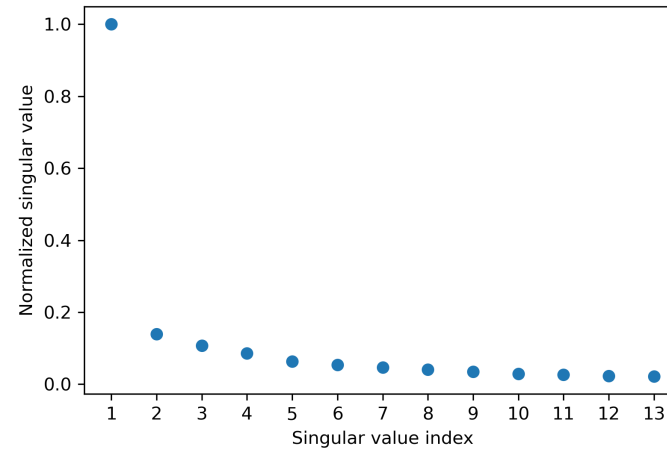


# Nunavik's flag is more complex



⇒ Good low-rank approximation

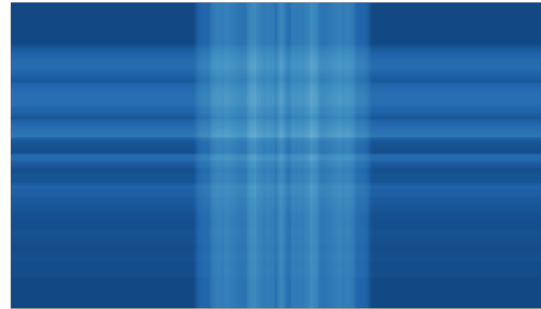
# Nunavik's flag is more complex



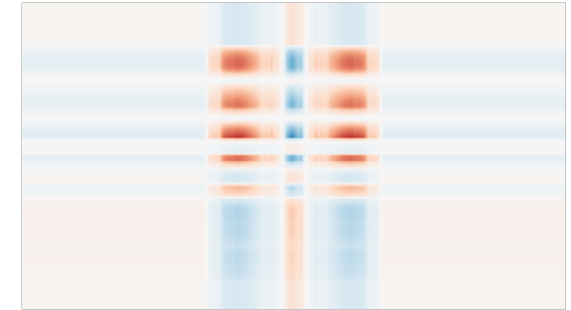
⇒ Good low-rank approximation



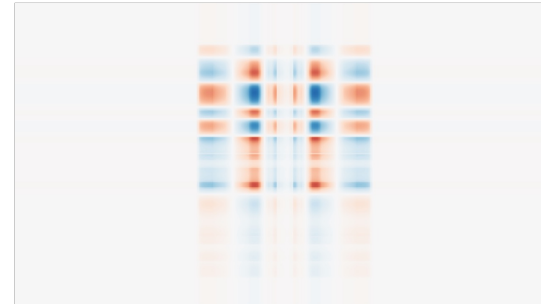
=



+

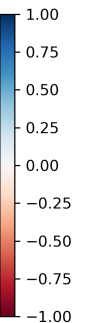


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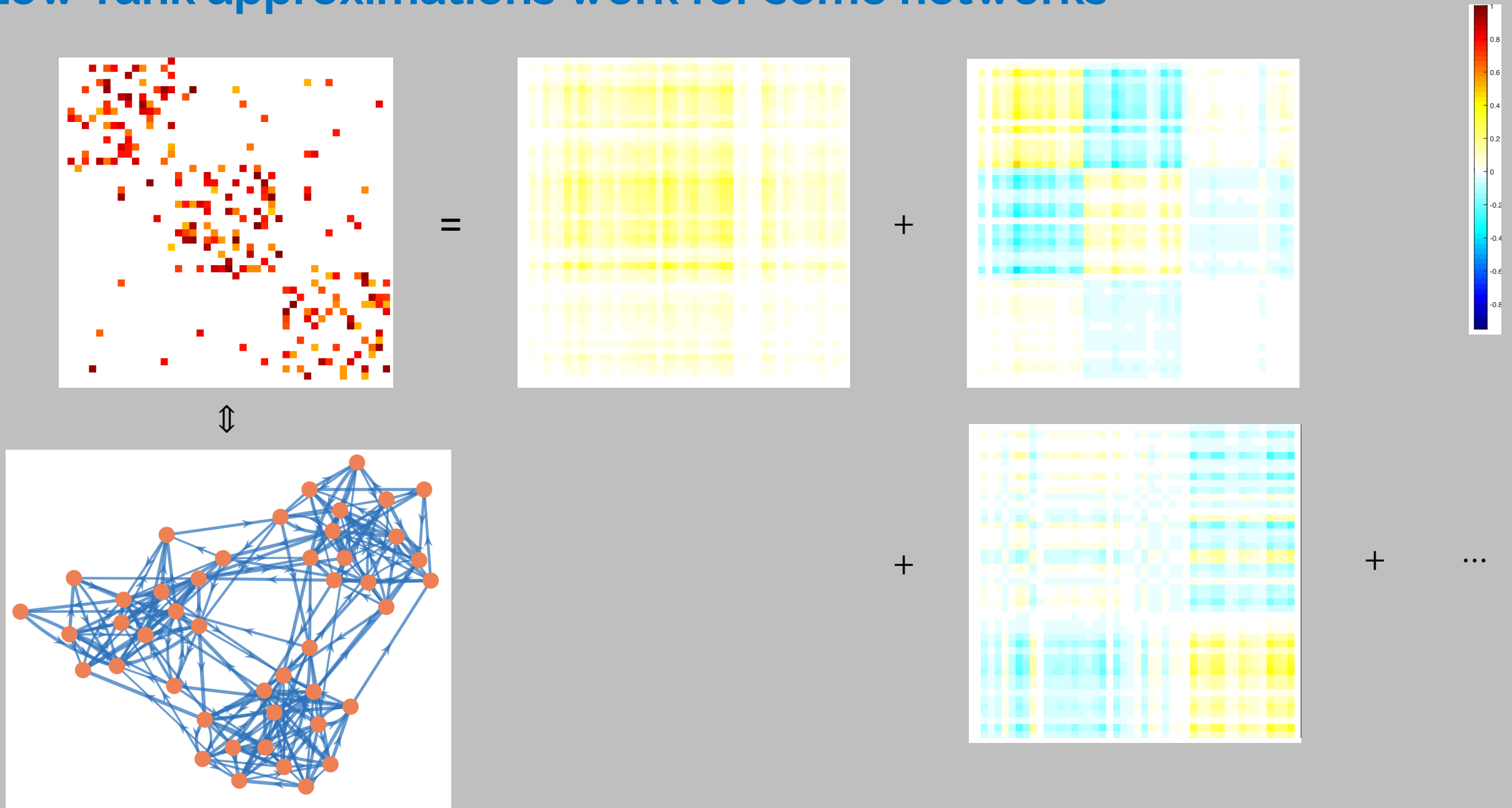


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...



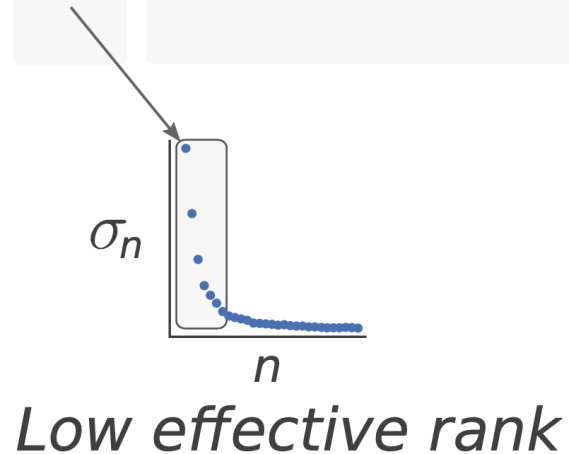
# Low-rank approximations work for some networks



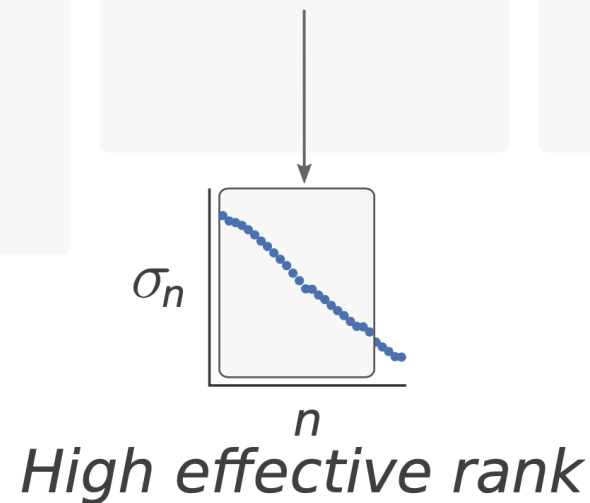
# Fundamental notion: effective rank

Effective rank of  $W$  = number of significant singular values of  $W$

Example: the stable rank defined as  $\text{srank}(W) = \frac{\sum_{i=1}^N \sigma_i^2}{\sigma_1^2}$



or

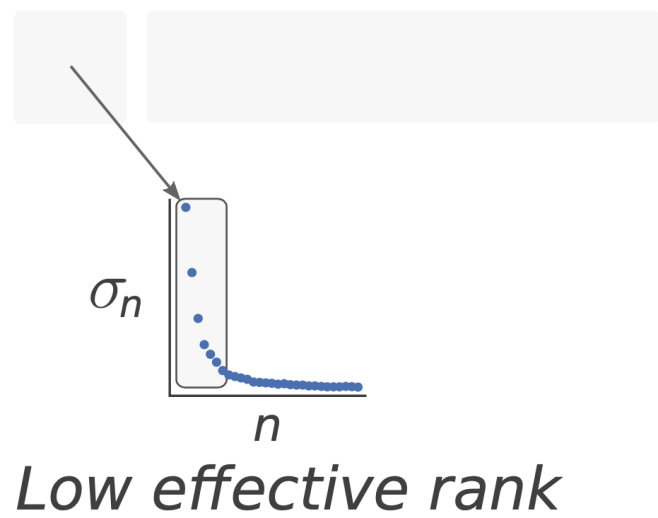


?

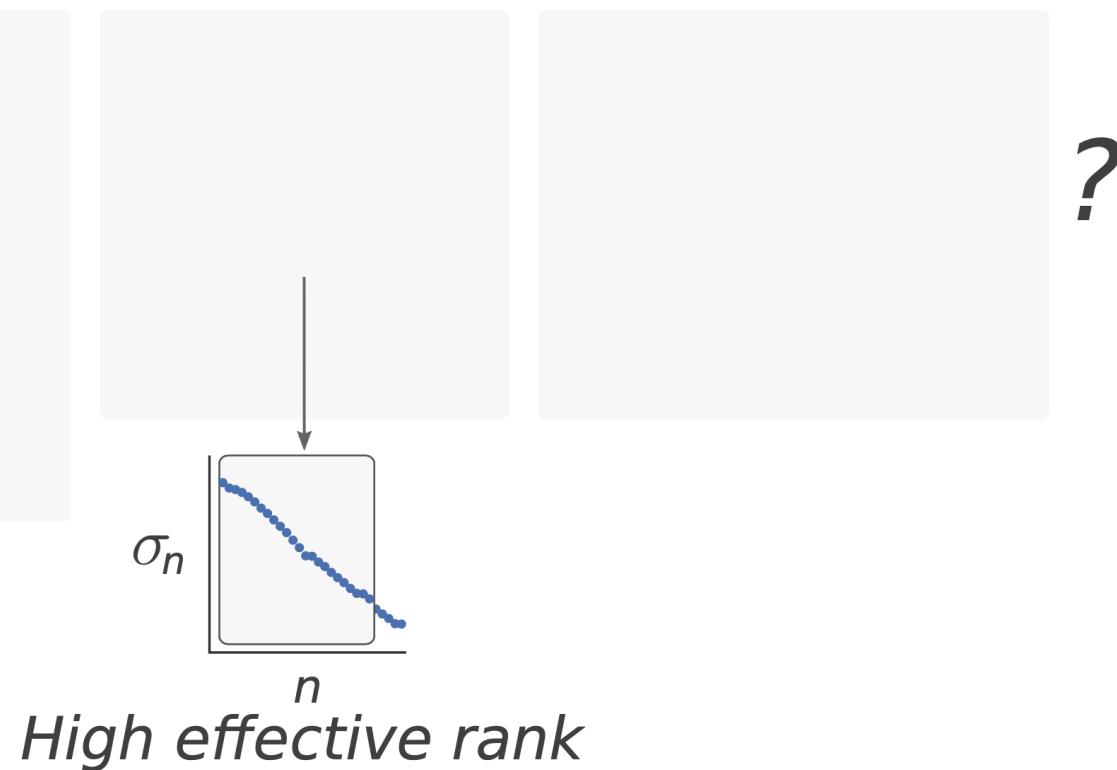


# Fundamental notion: effective rank

Effective rank of  $W$  = approximate dimension of the space generated by all  $Wx$

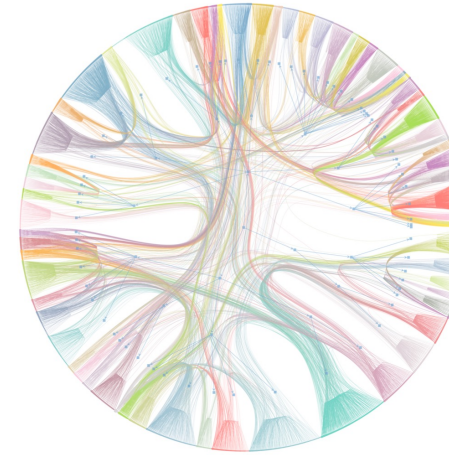
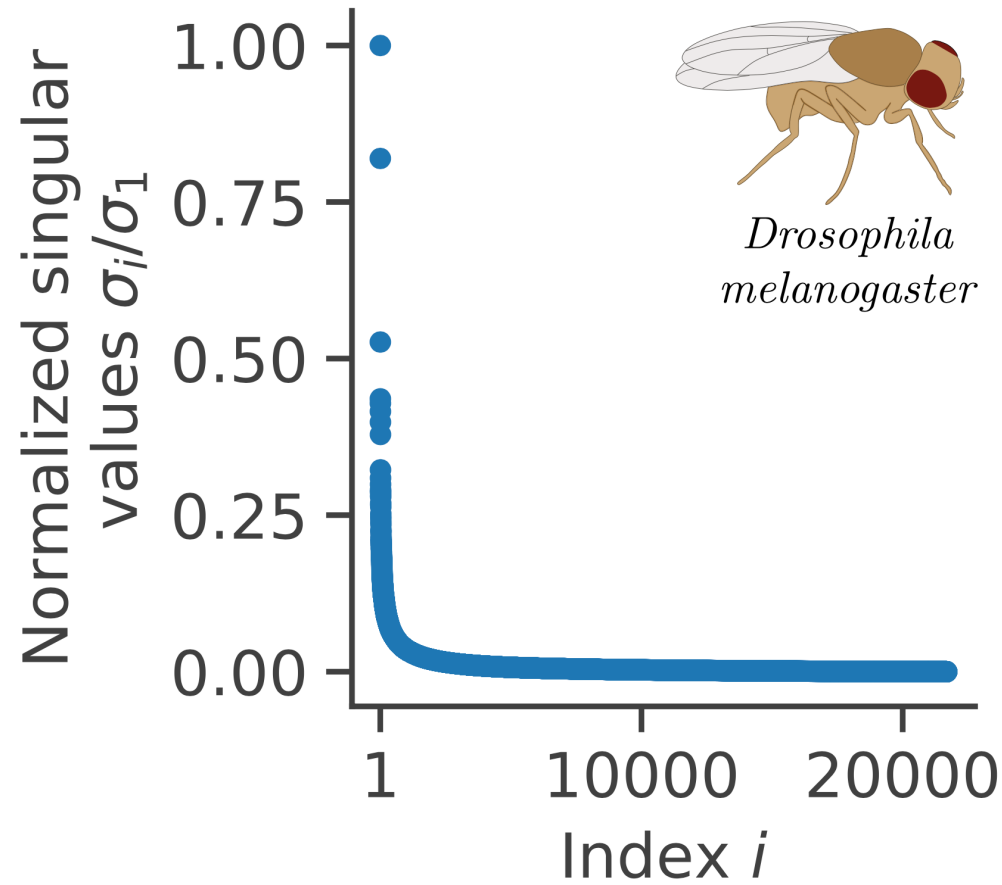


or



# Experimental results:

## Connectomes have low effective rank



eLife

RESEARCH ARTICLE

### A connectome and analysis of the adult *Drosophila* central brain

Louis K Scheffer<sup>1</sup>\*, C Shan Xu<sup>1</sup>, Michal Januszewski<sup>2</sup>, Zhiyuan Lu<sup>1,3,4</sup>, Shin-ya Takemura<sup>1</sup>, Kenneth J Hayworth<sup>1</sup>, Gary B Huang<sup>1</sup>, Kazunori Shinomiya<sup>1</sup>, Jeremy Maitlin-Shepard<sup>1</sup>, Stuart Berg<sup>1</sup>, Jody Clements<sup>1</sup>, Philip M Hubbard<sup>1</sup>, William T Katz<sup>1</sup>, Lowell Umayam<sup>1</sup>, Ting Zhao<sup>1</sup>, David Ackerman<sup>1</sup>, Tim Blakely<sup>2</sup>, John Bogovic<sup>1</sup>, Tom Dolafi<sup>1</sup>, Dagmar Kainmueller<sup>1,5</sup>, Takashi Kawase<sup>1</sup>, Khaled A Khairy<sup>1,6</sup>, Laramie Leavitt<sup>2</sup>, Peter H Li<sup>2</sup>, Larry Lindsey<sup>2</sup>, Nicole Neubarth<sup>1</sup>, Donald J Olbris<sup>1</sup>, Hideo Otsuna<sup>1</sup>, Eric T Trautman<sup>1</sup>, Masayoshi Ito<sup>1,7</sup>, Alexander S Bates<sup>1</sup>, Jens Goldammer<sup>1,7</sup>, Tanya Wolff<sup>1</sup>, Robert Svirskas<sup>1</sup>, Philipp Schlegel<sup>1</sup>, Erika Neace<sup>1</sup>, Christopher J Knecht<sup>1</sup>, Chelsea X Alvarado<sup>1</sup>, Dennis A Bailey<sup>1</sup>, Samantha Ballinger<sup>1</sup>, Jolanta A Borycz<sup>1</sup>, Brandon S Canino<sup>1</sup>, Natasha Cheatham<sup>1</sup>, Michael Cook<sup>1</sup>, Marisa Dreher<sup>1</sup>, Octave Duclos<sup>1</sup>, Bryon Eubanks<sup>1</sup>, Kelli Fairbanks<sup>1</sup>, Samantha Finley<sup>1</sup>, Nora Forknall<sup>1</sup>, Audrey Francis<sup>1</sup>, Gary Patrick Hopkins<sup>1</sup>, Emily M Joyce<sup>1</sup>, SungJin Kim<sup>1</sup>, Nicole A Kirk<sup>1</sup>, Julie Kovalyak<sup>1</sup>, Shirley A Lauchie<sup>1</sup>, Alanna Lohff<sup>1</sup>, Charli Maldonado<sup>1</sup>, Emily A Manley<sup>1</sup>, Sari McLin<sup>1</sup>, Caroline Mooney<sup>1</sup>, Miatta Ndama<sup>1</sup>, Omotara Ogundeyi<sup>1</sup>, Nneoma Okeoma<sup>1</sup>, Christopher Ordish<sup>1</sup>, Nicholas Padilla<sup>1</sup>, Christopher M Patrick<sup>1</sup>, Tyler Paterson<sup>1</sup>, Elliott E Phillips<sup>1</sup>, Emily M Phillips<sup>1</sup>, Neha Rampally<sup>1</sup>, Caitlin Ribeiro<sup>1</sup>, Madelaine K Robertson<sup>1</sup>, Jon Thomson Rymer<sup>1</sup>, Sean M Ryan<sup>1</sup>, Megan Sammons<sup>1</sup>, Anne K Scott<sup>1</sup>, Ashley L Scott<sup>1</sup>, Aya Shinomiya<sup>1</sup>, Claire Smith<sup>1</sup>, Kelsey Smith<sup>1</sup>, Natalie L Smith<sup>1</sup>, Margaret A Sobeski<sup>1</sup>, Alia Suleiman<sup>1</sup>, Jackie Swift<sup>1</sup>, Satoko Takemura<sup>1</sup>, Iris Talebi<sup>1</sup>, Dorota Tarnogorska<sup>1</sup>, Emily Tenshaw<sup>1</sup>, Temour Tokhi<sup>1</sup>, John J Walsh<sup>1</sup>, Tansy Yang<sup>1</sup>, Jane Anne Horne<sup>1</sup>, Feng Li<sup>1</sup>, Ruchi Parekh<sup>1</sup>, Patricia K Rivlin<sup>1</sup>, Vivek Jayaraman<sup>1</sup>, Marta Costa<sup>1</sup>, Gregory SXE Jefferis<sup>4,8</sup>, Kei Ito<sup>1,8,7</sup>, Stephan Saalfeld<sup>1</sup>, Reed George<sup>1</sup>, Ian A Meinertzhagen<sup>1,9</sup>, Gerald M Rubin<sup>1</sup>, Harald F Hess<sup>1</sup>, Viren Jain<sup>1</sup>, Stephen M Plaza<sup>1\*</sup>

\*For correspondence: [scheffer@janelia.hhmi.org](mailto:scheffer@janelia.hhmi.org) (LKS); [plaza@janelia.hhmi.org](mailto:plaza@janelia.hhmi.org) (SMP)

<sup>1</sup>These authors contributed equally to this work

Present address: <sup>1</sup>Max Delbrück Centre for Developmental Medicine, Berlin, Germany; <sup>2</sup>Department of Developmental Neurobiology, St. Jude Children's Research Hospital, Memphis, United States; <sup>3</sup>Two Six Labs, Arlington, United States

Competing interest: See page S2

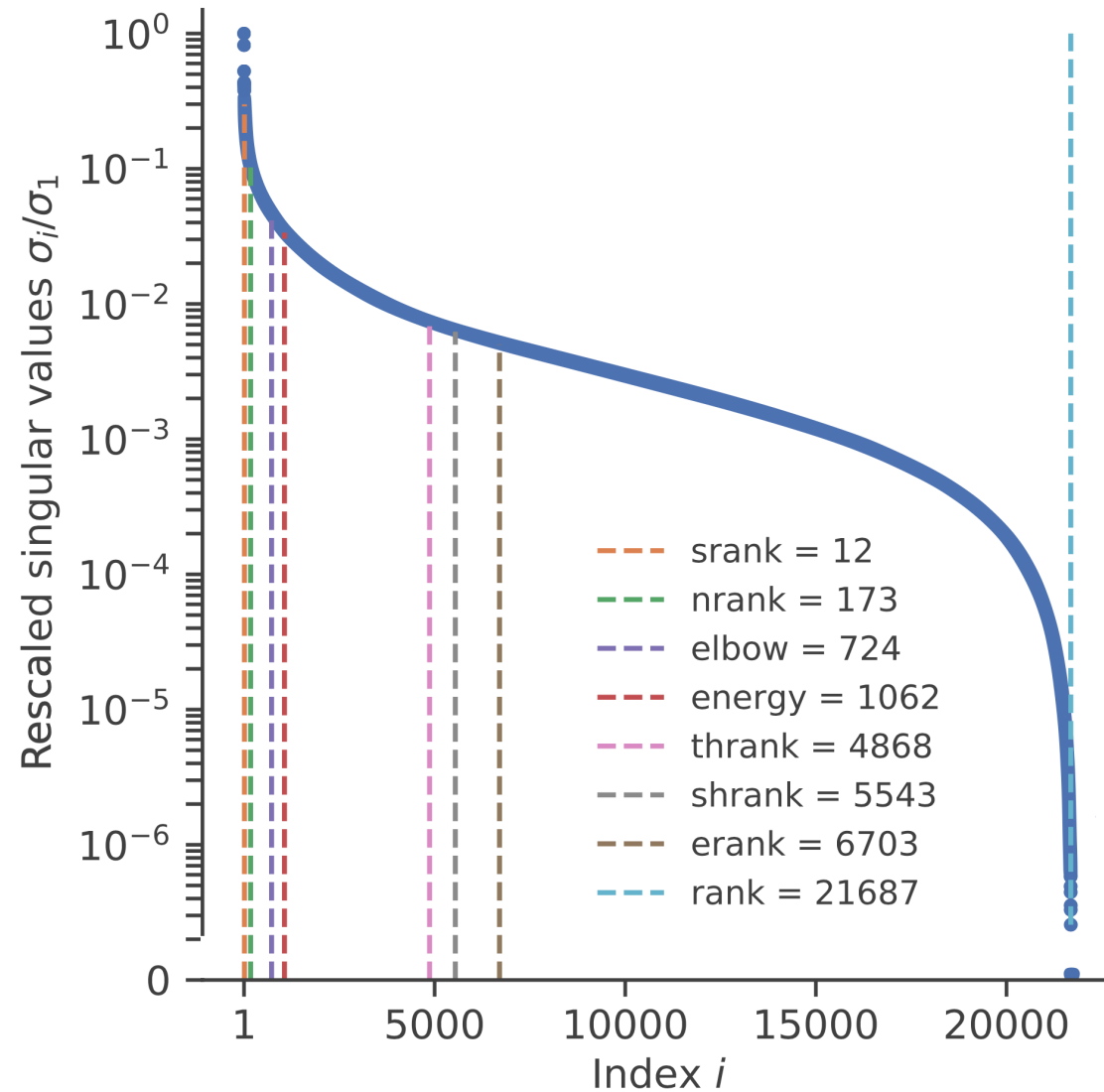
Funding: See page S2

Received: 31 March 2020  
Accepted: 01 September 2020  
Published: 07 September 2020

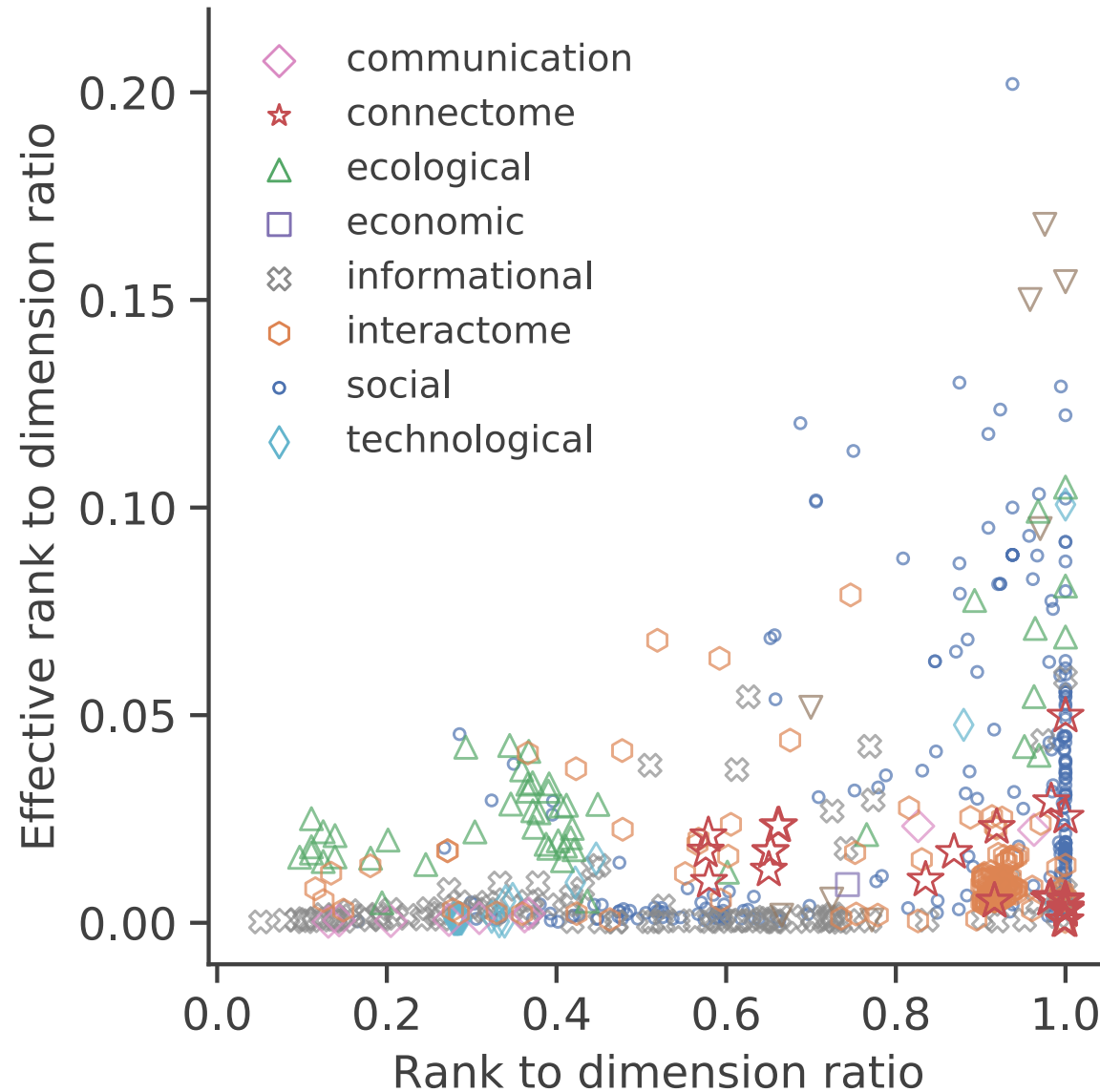
<sup>1</sup>Janelia Research Campus, Howard Hughes Medical Institute, Ashburn, United States; <sup>2</sup>Google Research, Mountain View, United States; <sup>3</sup>Life Sciences Centre, Dalhousie University, Halifax, Canada; <sup>4</sup>Google Research, Google LLC, Zurich, Switzerland; <sup>5</sup>Institute for Quantitative Biosciences, University of Tokyo, Tokyo, Japan; <sup>6</sup>MRC Laboratory of Molecular Biology, Cambridge, United States; <sup>7</sup>Institute of Zoology, Biocenter Cologne, University of Cologne, Cologne, Germany; <sup>8</sup>Department of Zoology, University of Cambridge, Cambridge, United Kingdom

# Experimental results:

## Connectomes have low effective rank



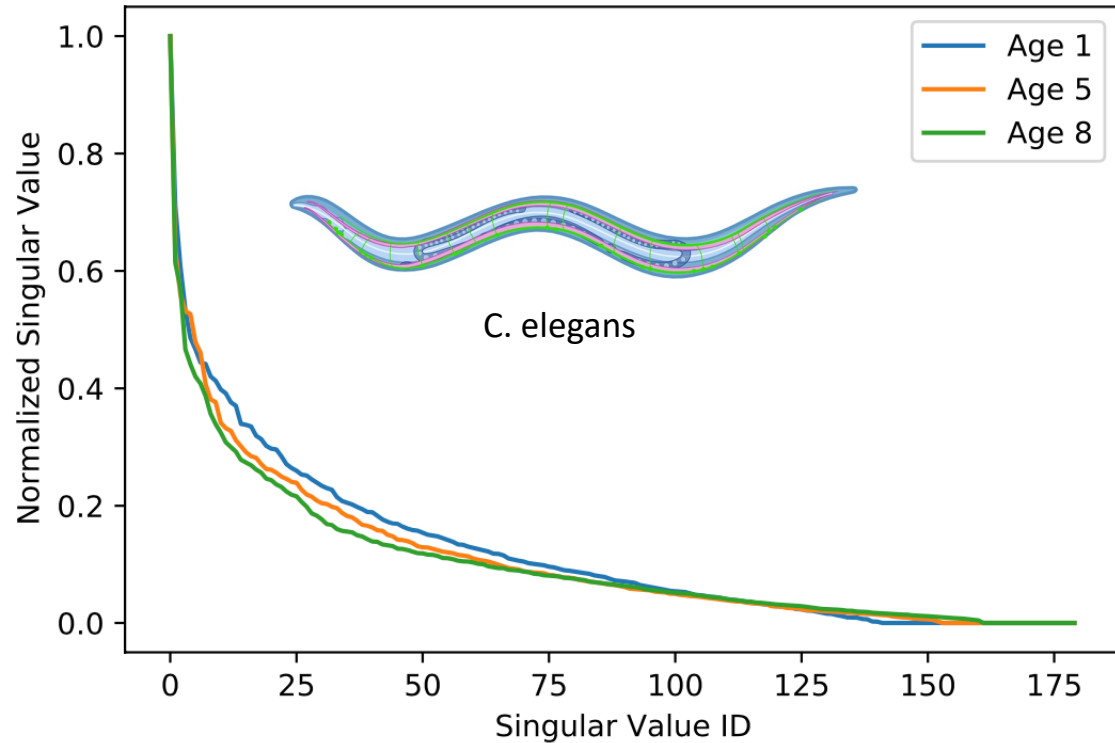
# Experimental results: real complex networks have low effective rank





# Observation:

## Maturation seems to reduce effective rank



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### Connectomes across development reveal principles of brain maturation

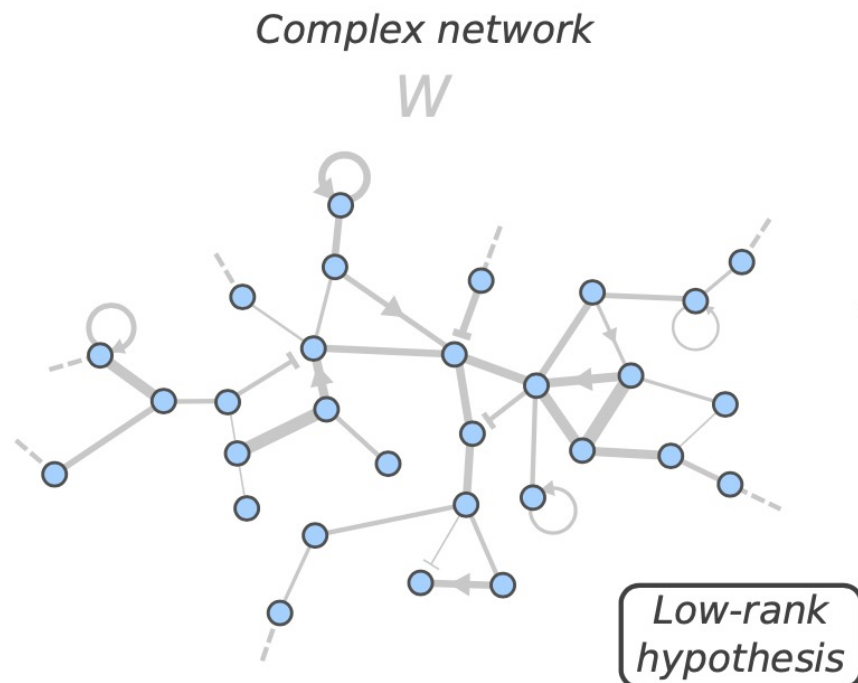
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[Nature](#) **596**, 257–261 (2021) | [Cite this article](#)

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Singular values of the matrices describing the connectivity of the *C. elegans* brain at different maturation stages. The stable ranks are 21.6 (age 1), 19.7 (age 5), 18.5 (age 8).

**What are the dynamical consequences of  
low effective rank ?**

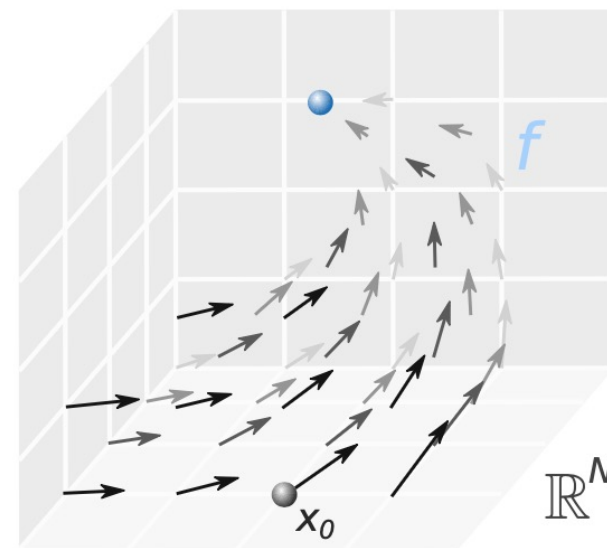


Vector field  $f$

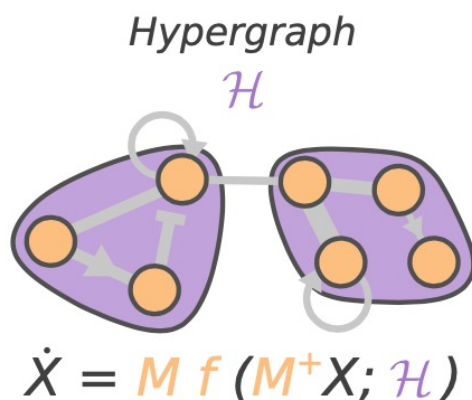


High-dimensional dynamics

$$\dot{x} = f(x; W)$$



Emergence of higher-order interactions



Low-dimension hypothesis

Reduction matrix  $M$

Optimal  $M$  determined by the weight matrix  $W$

Optimal vector field

$$G = M \circ f \circ M^+$$



$$\dot{X} = G(X; \text{Structure?})$$

Low-dimensional dynamics

Complete dynamics :  $\dot{x} = f(x)$

Reduced dynamics :  $\dot{X} = G(X)$  where  $X = Mx$

### THEOREM (SIMPLIFIED)

*The vector field  $G^*$  that minimizes the quadratic error between the projected dynamics  $\dot{p} = f(p)$  with  $p = M^+ Mx$  and the reduced dynamics in  $\mathbb{R}^N$   $[M^+ G(X)]$  is*

$$G^*(X) = M f(M^+ X).$$

*Proof* : Just use least-squares.

### THEOREM (SIMPLIFIED)

*The alignment error  $\mathcal{E}_f(x)$  for some  $x \in \mathbb{R}^N$  is upper-bounded by*

$$\mathcal{E}_f(x) \leq \frac{1}{\sqrt{n}} \left[ \|V_n^\top J_x(x', y')(I - V_n V_n^\top)x\| + \frac{\sigma_{n+1}}{\sigma_1} \|V_n^\top J_y(x', y')\|_2 \|x\| \right].$$

$\sigma_i$  :  $i$ -th singular values of  $W$

$V_n$  :  $n$ -truncated right singular vector matrix

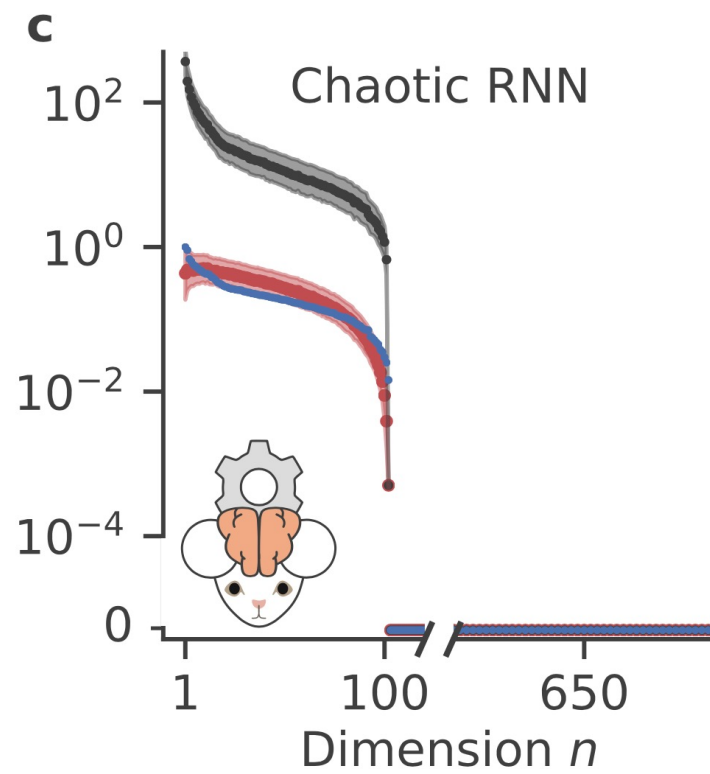
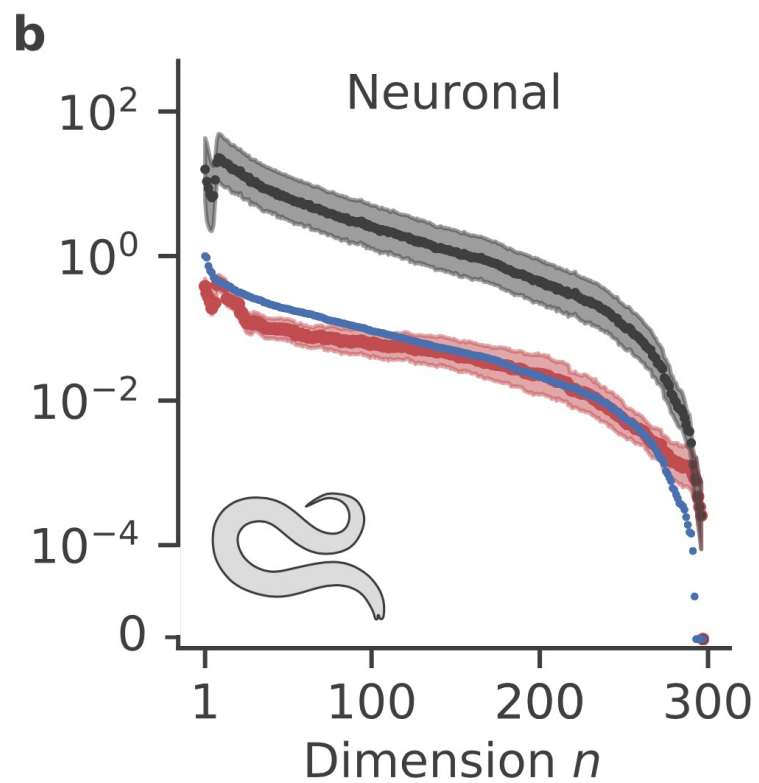
$J_x, J_y$  : Jacobian matrices evaluated at some point  $x', y'$

$n$  : dimension of the reduced system

$J_x(x', y') = aI$  and  $n \geq \text{rank}(W)$   $\Rightarrow$  Exact dimension reduction



••• Average alignment error  $\langle \mathcal{E} \rangle$     
 ••• Average upper-bound on  $\mathcal{E}(x)$     
 ••• Rescaled singular values  $\frac{\sigma_n}{\sigma_1}$



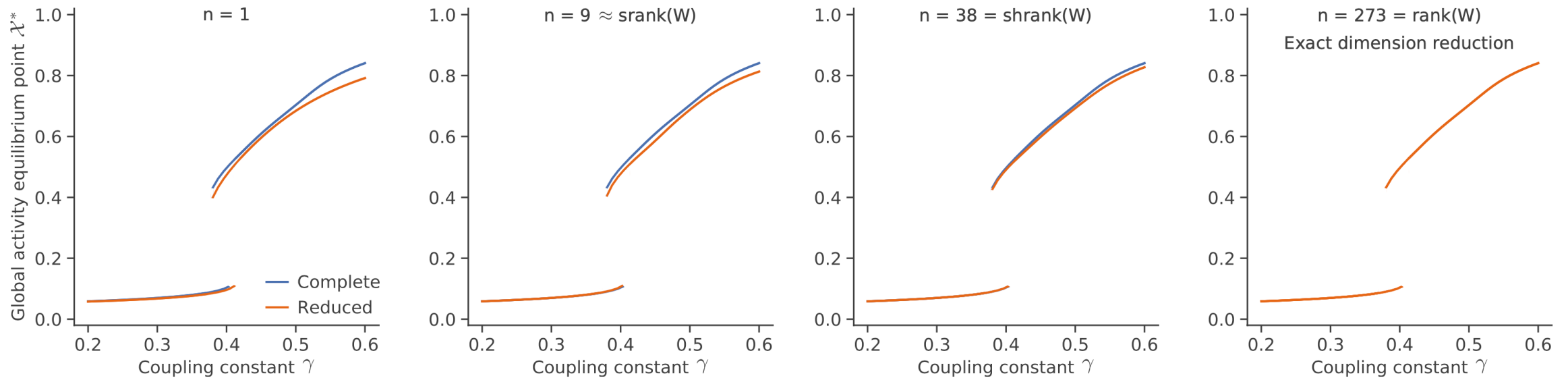


Fig. S8: Comparison between the global observable at equilibrium  $\mathcal{X}^*$  of the complete (blue) and reduced (orange) Wilson-Cowan dynamics on the (unsigned) *C. elegans* connectomes ( $N = 279$ ,  $\text{rank}(W) = 273$ ) vs. the global coupling  $\gamma$  for  $n \in \{1, 9, 38, 273\}$ . Parameters:  $d = 1$ ,  $a = 0$ ,  $b = 1$ ,  $c = 3$ . For the weight matrix, see the [GitHub repository](#), module `get_real_network.py`, function `get_connectome_weight_matrix(graph_name="celegans")`. The effective ranks of this connectome with weight matrix  $W$  are  $\text{srnk}(W) \approx 9$ ,  $\text{thrank}(W) = 27$ ,  $\text{elbow}(W) = 31$ ,  $\text{nrnk}(W) \approx 36$ ,  $\text{shrank}(W) = 38$ ,  $\text{energy}(W) = 106$ , and  $\text{erank}(W) \approx 192$ .

# Take-home messages

1. Whole brain neuronal activity can be modeled using firing rate models, which are high dimensional.
2. Real networks, and especially *connectomes*, have *low effective rank*.
3. Large neuronal networks with *firing rate dynamics possess low-dimensional dynamical systems* that approximately describe the activity at large scale.
4. Alignment errors of reduced vector fields can rapidly decrease following the singular values of complex networks.
5. Our theoretical findings support the use of Principal Component Analysis when analyzing neuronal activity.
6. Dimension reduction can lead to dynamics with *higher-order interactions*.

Still so much work to do ...

# Thank you! Questions?

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Understanding the structure and the dynamics of complex systems.



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### DOCTORAL STUDENTS



Antoine Légaré



Simon Lizotte



Charles Murphy



Marziyeh Pourmousavi



Jérémie Gince



Jeson Hermans



Jérémie Lesage

### INTERNS



Émile Baril



Benjamin Claveau



Anthony Drouin

### MASTER STUDENTS

### GROUP LEADERS



Antoine Allard



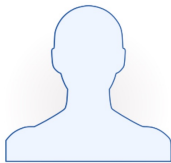
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### HONORARY MEMBER



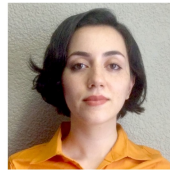
Louis J. Dubé



François Thibault



Vincent Thibeault



Zahra Yazdani



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