Phase space as an optical engineering tool in open microcavity designs
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Physical set-up
Dielectric microcavities have attracted considerable attention recently due to promising technological advances in sensing and laser applications [1]. These resonators consist in a thin slab of dielectric material whose boundaries’ geometry and refractive index define the behaviour of the supported light field (Figs. 1 and 2). Light is emitted in the far-field.

Wave / Classical dynamics
For a monochromatic electric field normal to the plane of a thin cavity, the corresponding scalar electric field ψ(r) is solution of Helmholtz’s wave equation

\[ \nabla^2 \psi + k^2 \psi = 0, \quad r \in \mathbb{R}^2 \]  

(1)

- \( n(r) \): refractive index; \( k \): wavenumber
- BCs: Continuity of the field and its derivative
- Sol’n: Scattering matrix formalism.

Setting \( k \rightarrow \infty \) leads to semi-classical limit:

- Spectral reflection at boundary
- Refraction: \( n_\text{in} \sin \chi = n_\text{out} \sin \chi_\text{r} \)
- Total Internal Reflection (TIR) when

\[ |\sin \chi| > n_\text{in}/n_\text{R} \]  

(2)

- BILLIARD system: Manifolnd dynamics
- Poincaré section on external boundary
- Canonical coordinates: \( s \) (arc-length along boundary) and \( p = \sin \chi \) (linear momentum).

Bridging the gap
Correspondence between classical and wave dynamics may be drawn using Husimi’s distribution \( F^H(s,p) \) [2]:

- \( F^H(s,p) \) associates \( \psi(s) \) with a distribution in phase space
- Canonical coordinates \( s,p \) on Poincaré section

A paradigm for mixed system: The annular cavity
Because of the seemingly impossibility to meet both high directionality and storage capacity requirements, we consider mixed dynamics systems supporting both regular regions and a “chaotic sea”. One particular member of this large set of systems is the annular cavity [3] (Fig. 6).

Harnesing the power flow: Parametric control
Figs. 8-9 show that it is possible to trigger anisotropic emission. We can now “control” the emission by modifying hole radius \( s \) while keeping \( d = \text{const} \). (\( d = 0.7 \))

- Set of initial conditions around \( |s| = (d + a)/R \)
- \( F^H(s,p) \) of mode (27,3) is restricted to emission region.

Extreme scenarios
The properties of the field contained in dielectric cavities strongly depend on the boundary geometry:

- Disc cavity (Fig. 4): Non-directional emission / High energy containment
- Completely regular phase space
- Sol’n of eq. (1) inside cavity: \( \psi_\text{in} \sim J_0(\alpha_\text{in} kr) \cos \theta \)
- Husimi’s distribution is a gaussian function centered at \( p = \alpha_\text{in} k R \) for \( s = R \).

- Stadium cavity (Fig. 5, \( k > 0 \)):
- Highly directional emission / Poor energy containment
- Completely chaotic phase space
- Light emission guided by unstable manifolds [3].

Fig 6: Schematic view of the annular cavity. A “hole” with refractive index \( n_\text{in} \) is inserted in a disc cavity: annular space.

Fig 7: Regions of phase space associated with regular and “chaotic” trajectories. Inset: Corresponding domain in (s,p) space.

Fig 8: Extremes of \( F^H(s,p) \) for \( k = 8, \alpha_\text{in} = 0.1, R = 1, s = 0.2 \); the chaotic region is embedded inside an annular region: Right, isotropic emission in the far-field; “chaotic” modes (27,3) and resonance level remain mostly unaffected.

Fig 9: Phase space with \( F^H(s,p) \) and far-field emission for \( a = 0.1 \) (top), \( a = 0.5 \) (middle) and \( a = 0.3 \) (bottom).

- Classical emission: Extension of unstable manifolds in emission region
- \( F^H(s,p) \) follow unstable manifolds: Emission modified.

Through the modification of phase space, we may then

- Induce anisotropic emission of the disc’s modes and
- Modify the far-field emission patterns
- While keeping high energy storage levels.