Persistent activity of neural dynamics on hierarchical networks

Edward Laurence¹, Patrick Desrosiers¹,², and Louis J. Dubé¹

¹ Département de physique, de génie physique, et d’optique, Université Laval, Québec, Canada
² Centre de recherche CERVO, Québec, Canada

Summary

Hierarchy has been hypothesized to facilitate the emergence of persistent activity [1]. We explore this statement with different hierarchical organizations on a binary neural dynamics. The multiple stable levels of activity, called persistent activity, are studied analytically and numerically.

- We analytically bounded the number of intermediate states of activity for random, modular and hierarchical directed structures.
- Hierarchical structures reduce the diversity of activities compared to modular structures.
- For HPA structures, persistent activity emerges in a narrow window of parameters and shows delayed activation.

Model

Binary dynamics with spontaneous activation

Consider a graph composed of \( N \) neurons of binary activity \( X_i(t) \) at time \( t \) and input activity \( m_i(t) = \sum_j w_{ij} X_j(t) \).

Rate of activation \( \frac{1}{1+\nu} \)

Rate of inactivation \( \lambda \)

Master equation

\[
\mathbb{E}\{X_i(t+\Delta t)\} = \mathbb{E}\{X_i(t)\} \left(1 + \lambda \sum_{j} w_{ij} X_j(t)\right) - \mathbb{E}\{m_i(t)\}\Delta t + \mathbb{E}\{X_i(t)\}\Delta t
\]

The master equation for the probability of being active is

\[
P(A(t+\Delta t)|A(t)) = P(A(t)) \left(1 - \frac{\lambda}{1+\nu} \right) + P(A(t)) \left(1 - \frac{\lambda}{1+\nu}\right) \Delta t
\]

Each node has at least one and at most two stable steady-states.
- Spontaneous activation enables a minimal level of activity.

Persistent activity

Persistent activity has been introduced by Kaiser et al. [1], to describe intermediate states of activity detected in neural networks.
- Persistent activity is an intermediate and stable state of activity.

In modular networks, we have found that persistent activity emerges from a weak communication between structures.

Bounds on the activity (\( \lambda = 0 \))

Random network

Let us consider a random network of density \( p \). To be active, a node must have a number of active neighbors larger than a certain threshold parameter \( \mu \). A mean-field analysis provides two constraints on the number of active nodes \( r \).

- Inactive nodes \( \frac{p r}{1+\nu} < \mu \)
- Active nodes \( \frac{p (r-1)}{1+\nu} \geq \mu \)

Leading to

\[
\frac{\mu (1+\nu)}{p} \leq r \leq \frac{\mu (1+\nu)}{p}
\]

- Two stable solutions exist: nodes are either all active or all inactive.

Modular network

Let \( r \) be the number of active nodes from a planted partition of densities \( p_a \) and \( p_v \) and communities of size \( n \).

- Inactive nodes \( \frac{p a r}{1+\nu} < \mu \)
- Active nodes \( \frac{p (r-1)n}{1+\nu} \geq \mu \)

It leads to boundaries on the number of active nodes.

- The number of active nodes is highly constrained by the dynamics and the densities of the network.
- Persistent activity of modular networks is bounded.

Hierarchical network

Let us consider a structure of 2 hierarchical levels of organization and \( k \) structures at the highest level and \( k \) communities of size \( n \) at the lowest level. Densities between structures are \( p_{a1} \) and \( p_{a2} \) (see figure on the left).

- The modular structure to obtain constraints on the number of active nodes.

- For equivalent networks, hierarchical structures have smaller regions of possible number of active nodes than modular structures.
- Hierarchy reduces the diversity of persistent activities.

HPA structures

Hierarchical preferential attachment (HPA)

HPA has been introduced in Ref. [2] and is characterized by
- Scale-free degree distribution.
- Scale-free community size and membership distributions.
- Realistic model for hierarchical structures.

Observation of persistent activity

We have applied the binary dynamics on HPA networks of four hierarchical levels. We have controlled the structure using a parameter \( \beta \).
- Intermediate states of activity emerge close to the critical threshold of activation.
- The transition to endemic state is delayed.

Example of the bounds for the number of active nodes.

- Active nodes of hierarchical planted partitions.
- Modular bounds
- Hierarchical bounds

Bibliography


Acknowledgements

Edward Laurence1, Patrick Desrosiers1,2, and Louis J. Dubé1

1 Département de physique, de génie physique, et d’optique, Université Laval, Québec, Canada
2 Centre de recherche CERVO, Québec, Canada