Controlling transport properties in dielectric billiards

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Context

Dynamical tunneling in quantum systems of more-than-one-degree-of-freedom exhibits many features lacking from the well-known one-degree-of-freedom tunneling. One of those new phenomena is Chaos-Assisted Tunneling (CAT) which occurs when the interaction between two "regular" states is enhanced by the proximity of a third "chaotic" state.

Out of the many fields that will benefit improvements in understanding this phenomenon, optics is one that could lead to important technological advances. We propose a simple physical model to study CAT in optical microcavities.

Interaction models

When states of a given system interact by dynamical tunneling, their eigenvalue generally exhibits avoided-crossings through parametric modification. To understand this phenomenon, one can model the system near the interaction with a local interaction matrix. Two important cases are presented below: direct tunneling and CAT.

Direct tunneling

Interaction of 2 states $|E_1\rangle$ *and* $|E_2\rangle$

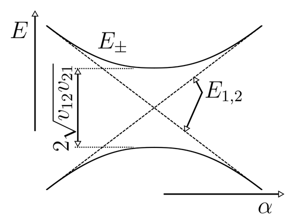


Fig. 1: Direct coupling between two locally isolated states.

- *Control parameter:* α
- *Unperturbed levels:* $E_1 = -E_2 = \alpha$
- *Local interaction matrix:*

$$\mathbf{M}_{Direct} = \begin{pmatrix} E_1(\alpha) & v_{12} \\ v_{21} & E_2(\alpha) \end{pmatrix}, \quad (1)$$

$v_{12} = v_{21}^*$*: coupling strength*

- *Energy levels:* $E_\pm = \pm\sqrt{\alpha^2 + v_{12}v_{21}}$, *anti-crossing.*

Chaos-Assisted Tunneling (CAT)

Interaction of 3 states: $|E_1\rangle$ *and* $|E_2\rangle$ *are "regular" and* $|E_C\rangle$ *is "chaotic"*

- *Regular states' control parameter:* α
- *Chaotic state's control parameter:* λ
- *Vanishing direct tunneling:* $v_{12} = v_{21} = 0$
- *Local interaction matrix:*

$$\mathbf{M}_{CAT} = \begin{pmatrix} E_1(\alpha) & 0 & v_{1c} \\ 0 & E_2(\alpha) & v_{2c} \\ v_{c1} & v_{c2} & E_c(\lambda) \end{pmatrix}, \quad (2)$$

$v_{jc} = v_{cj}^*$*: coupling strength*

- *Energy levels (perturbative sol'n): Regular levels' splitting increases with proximity of chaotic level.*

Tomsovic, S. J. Phys. A, 1998, 31, 9469-9481

Physical set-up

The physical problem of interest concerns the time-independent monochromatic electric field $\psi(\mathbf{r})$ *inside an optical microcavity. In this case, Maxwell's equations reduce to Helmholtz equation*

$$\left[\nabla^2 + n^2(\mathbf{r})k^2\right]\psi(\mathbf{r}) = 0, \quad (3)$$

- *Transverse magnetic polarization:* $\mathbf{E}(\mathbf{r}) = \psi(\mathbf{r})\hat{\mathbf{a}}_z$
- *Inhomogeneous refractive index:* $n(\mathbf{r})$
- *Wavenumber:* k.

Full-wave solution (finite k)

Closed cavity scenario ("perfectly conducting boundary")

- *Dirichlet BCs:* $\psi(\mathbf{r})|_{\partial\Omega} = 0$
- *Discrete set of eigenvalues:* $\{k_m\} \in \mathbb{R}$
- *Sol'n: Finite Element Method.*

Semiclassical limit

The limit $k \to \infty$ *leads to semi-classical dynamics:*

- *Birkhoff billiards: Specular reflection*
- *Quantification: Optical Path Length (OPL).*
- *Poincaré section on the boundary:* $(s, p = \sin\chi)$
- *Husimi distribution* $F^H(s,p)$*: A distribution in canonical coordinates associated with* $\psi(\mathbf{r})$*; establishes ray-wave correspondence*

Crespi, B. et. al. Rev. E, 1993, 47, 986-991

Inhomogeneous ellipse

The proposed geometry consists in an ellipse with constant refractive index containing two circular "holes", as Fig. 2 depicts. This system features many parameters:

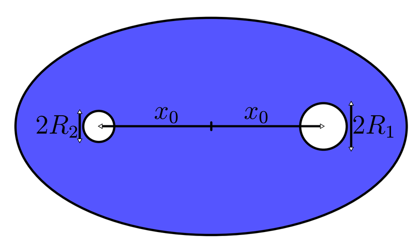


Fig. 2: ***Elliptic cavity featuring 2 holes. The whole structure may be regarded as an inhomogeneous cavity of integrable shape.***

Ellipse

- *Area:* A
- *Eccentricity:* ϵ
- *Refractive index:* n_0.

Two "holes"

- *Positions:* $x_1 = -x_2 \equiv x_0$
- *Radii:* $R_{1,2}$
- *Refractive indexes:* $n_{1,2} = 1$.

This geometry supports 3 distinct trajectory types

- *Regular with elliptic caustic (type 1)*
- *Regular with hyperbolic caustic (type 2)*
- *Chaotic.*

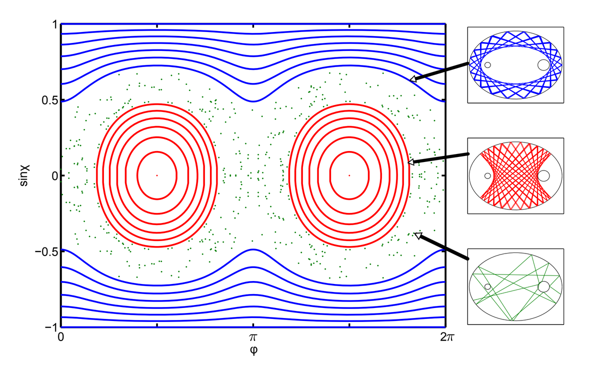


Fig. 3: ***Phase space for*** $A = \pi, \epsilon = 0.615, n_0 = 1.5, x_0 = 0.679, R_1 = 0.1414$ ***and*** $R_2 = 0.035$ ***and typical trajectory for the 3 regions of phase space.***

Control parameters

The different parameters are chosen such that the 3 regions of phase space are well separated (Fig. 3). One then considers the effects of the parameters on the interaction scenarios.

Direct coupling

The parameter ϵ *affects all state types. However, regarding the classical dynamics of the regular modes,* ϵ

- *Increases the OPL of type 1 trajectories*
- *Decreases the OPL of type 2 trajectories.*

One may then distinguish type 1 from type 2 by their parametric dependency. This behaviour will generate avoided-crossings between type 1 and type 2 states: $\epsilon \sim \alpha$.

CAT

Since the ellipse is y-axis symmetrical, modifying $R_2 \leq R_1$

- *Does not affect regular states (types 1 and 2)*
- *Affects only chaotic states (type 3).*

One may then consider matrix (2) as a valid model for local description near a 3 states interaction: $R_2 \sim \lambda$.

Controlling CAT follows a simple procedure:

1. *Find a weak coupling between 2 regular states (direct coupling) using* ϵ
2. *Steer a neighboring chaotic state close to the interaction region using* R_2.

Fig. 4 illustrates a desirable configuration of eigenvalues in order to observe and control CAT. This set-up is further analyzed in the next section.

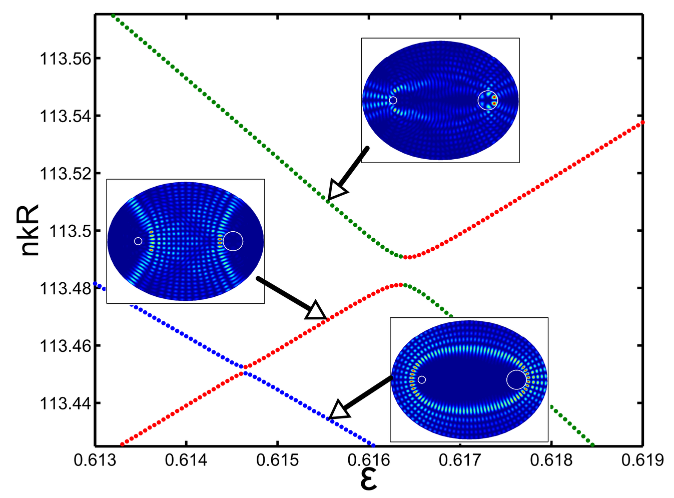
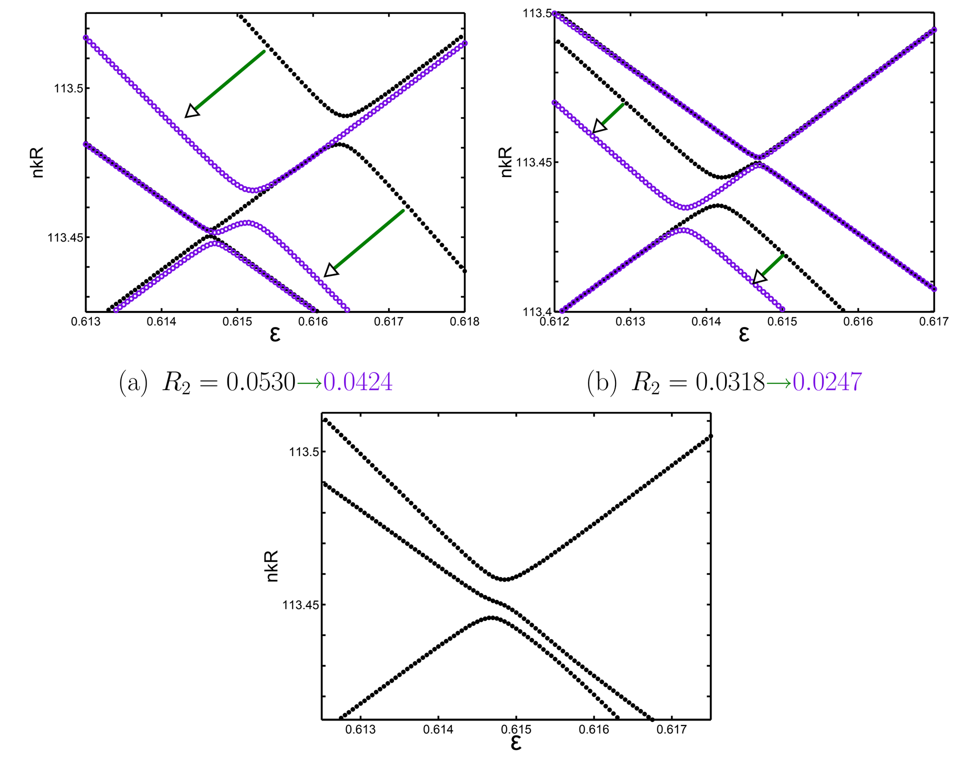


Fig. 4: ***Typical eigenvalue behaviour with regards to parameter*** ϵ ***near a 3 states interaction region. Husimi's distribution relates*** $\psi(\mathbf{r})$ ***to the corresponding phase space domain. Constant*** $R = \sqrt{A}$ ***is a characteristic length.***

CAT in action

The adiabatic behaviour of one chaotic state over a range of $R_2 \in [0.530, 0.247]$ *is investigated: the chaotic state intersects the avoided-crossing between two regular states. For CAT to occur, the splitting between the regular doublet should vary as the chaotic level approaches.*



(a) $R_2 = 0.0530 \to 0.0424$ (b) $R_2 = 0.0318 \to 0.0247$

(c) $R_2 = 0.0389$

Fig. 5: R_2 ***parametric dependency of the eigenstates of the triplet of Fig. 4. Each figure presents a different scenario : (a) the chaotic level approaches the regular doublet (b) the chaotic level leaves the regular doublet (c) all 3 levels are strongly interacting. For (a)-(b), two spectra are superimposed in order to show the levels' dynamics, the black dots identify the higher*** R_2 ***value.***

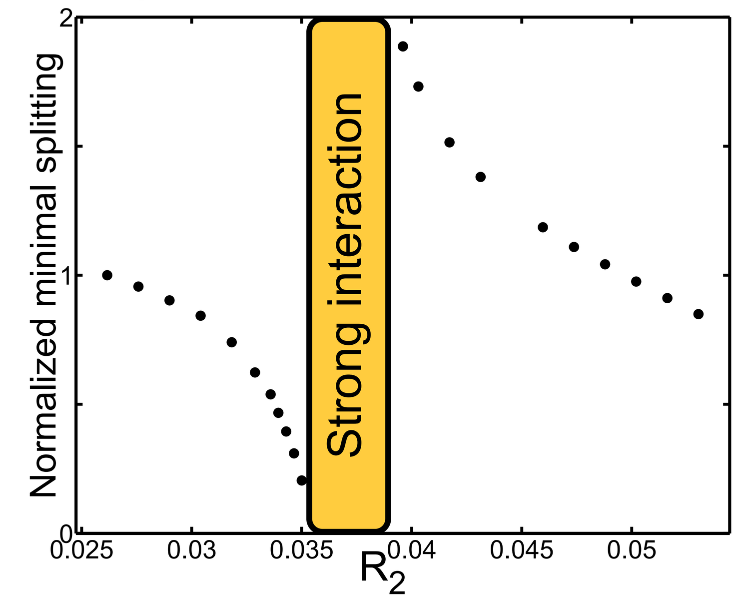


Fig. 6: ***The measured minimal splitting between the two regular states as the parameter*** R_2 ***is varied. This range of parameter is the same as shown on Fig. 5. For*** $\mathbf{R_2} \in [\mathbf{0.035}, \mathbf{0.039}]$***, those three modes are strongly interacting and the minimal splitting between the two regular states cannot be measured.***

Conclusion

- *A method to predict and control CAT in a real physical system has been presented.*
- *Dynamical tunneling connects different phase space regions.*
- *Same behaviour should appear for the equivalent open system. This could lead to control of directional emission.*