Phase space as an optical engineering tool in open microcavity designs

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Physical set-up

Dielectric microcavities have attracted considerable attention recently due to promising technological advances in sensing and laser applications [1]. These resonators consist in a thin slab of dielectric material whose boundaries' geometry and refractive index define the behaviour of the supported light field (Figs. 1 and 2). Light is emitted in the plane.

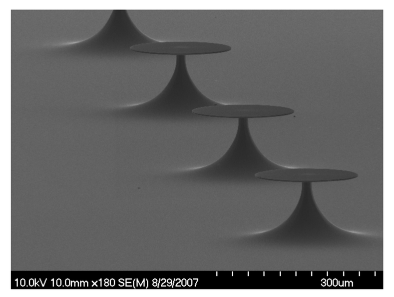


Fig. 1: ***Disc cavities (200μm diam.),*** *S. Saïdi, Polytechnique de Montréal.*

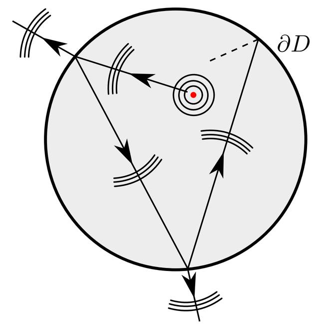


Fig. 2: ***Conceptual view of the leaky system: Partial containment of the light field by the boundary ∂D.***

A desirable set of goals to achieve is containment of high electromagnetic energy densities and directional emission of light. Since homogeneous disc cavities support large resonance modes but are bound to isotropic emission over all angles, we use a specific deformation of refractive index to modulate the emission while keeping strong resonance levels. Moreover, we propose to use the phase plane's structures to steer the light emission.

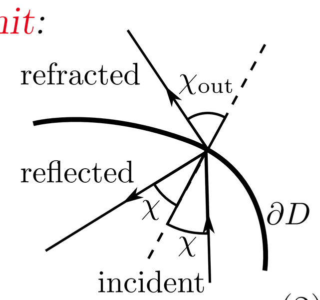
Wave / Classical dynamics

For a monochromatic electric field normal to the plane of a thin cavity, the corresponding scalar electric field $\psi(\mathbf{r})$ is solution of Helmholtz's wave equation

$$\left\{\nabla^2 + n^2(\mathbf{r})k^2\right\}\psi(\mathbf{r}) = 0,\ \mathbf{r}\in\mathbb{R}^2 \qquad (1)$$

- *$n(\mathbf{r})$: refractive index; k: wavenumber*
- *BCs: Continuity of the field and its derivative*
- *Sol'n: Scattering matrix formalism.*

Setting $k\to\infty$ leads to semi-classical limit:



- *Specular reflection at boundary*
- *Refraction: $n_{\text{in}}\sin\chi = n_{\text{out}}\sin\chi_{\text{out}}$*
- *Total Internal Reflection (TIR) when*

$$|\sin\chi| \geq n_{\text{out}}/n_{\text{in}} \qquad (2)$$

- *Billiard system: Hamiltonian dynamics*
- *Poincaré section on external boundary*
- *Canonical coordinates: s (arclength along boundary) and $p=\sin\chi$ (linear momentum).*

Bridging the gap

Correspondence between classical and wave dynamics may be drawn using Husimi's distribution $F^H(s,p)$ [2]:

- *$F^H(s,p)$ associates $\psi(\mathbf{r})$ with a distribution in phase space*
- *Canonical coordinates (s,p) on Poincaré section.*

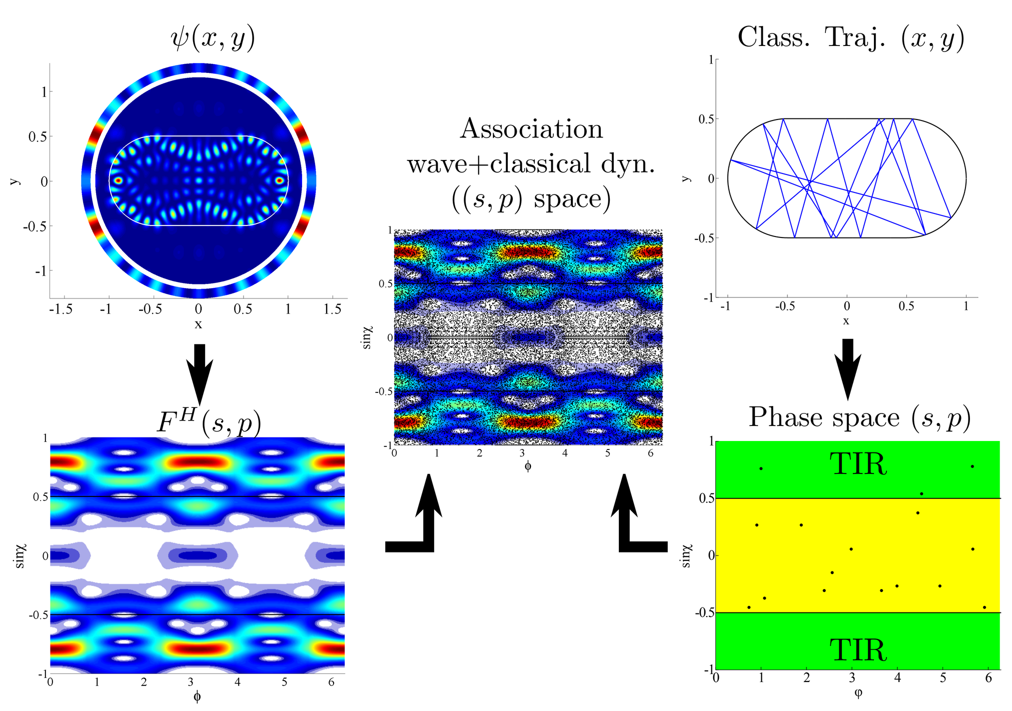


Fig. 3: ***Procedure followed to establish correspondence between classical and wave dynamics of the field contained inside the cavity; Stadium cavity, $n_{\text{in}}=2$ and $n_{\text{out}}=1$. Left column: (top) Solution of eq. (1) and (bottom) calculation of Husimi's distribution. Right column: (top) Typical trajectory and (bottom) corresponding phase space. Note the use of non-canonical coordinate ϕ instead of s and of the total field (incoming+scattered) on the left column. TIR limit, eq. (2), at $p=\pm0.5$.***

Extreme scenarios

The properties of the field contained in dielectric cavities strongly depend on the boundary geometry:

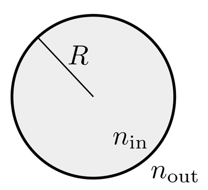


Fig. 4: ***Disc cavity.***

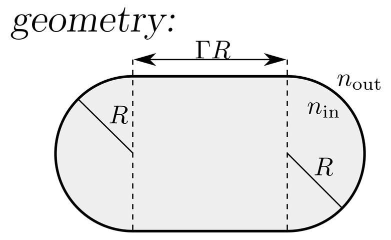


Fig. 5: ***Stadium cavity.***

- ***Disc cavity*** *(Fig. 4):*
  - *Non-directional emission / High energy containment*
  - *Completely regular phase space*
  - *Sol'n of eq. (1) inside cavity: $\psi_{\text{in}}\sim J_m(n_{\text{in}}kr)e^{im\phi}$*
  - *Husimi's distribution is a gaussian function centered at $p=m/nkR$ for all $s=R\phi$.*
- ***Stadium cavity*** *(Fig. 5, $\Gamma>0$):*
  - *Highly directional emission / Poor energy containment*
  - *Completely chaotic phase space*
  - *Light emission guided by unstable manifolds [3].*

A paradigm for mixed system: The annular cavity

Because of the seemingly impossibility to meet both high directionality and storage capacity requirements, we consider mixed dynamics systems supporting both regular regions and a "chaotic sea". One particular member of this large set of systems is the annular cavity [4] (Fig. 6).

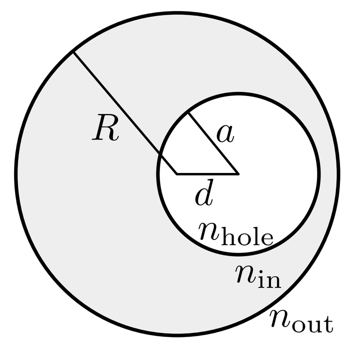


Fig. 6: ***Schematic view of the annular cavity. A "hole" with refractive index n_{hole} is inserted in a disc cavity. $n_{\text{in}}=3.2$ and $n_{\text{hole}}=n_{\text{out}}=1$.***

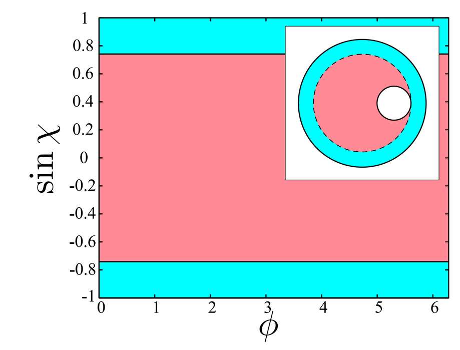


Fig. 7: ***Regions of phase space associated with regular and "chaotic" trajectories. Inset: Corresponding domains in (x,y) space.***

The trajectories intersecting the hole ("chaotic trajectories") are contained inside an area bounded by

$$|\sin\chi| \leq (d+a)/R \quad \forall\ s \quad . \qquad (3)$$

When $\psi(\mathbf{r})$ "fits" inside the blue region of the inset of Fig. 7, we expect:

- *$\psi(\mathbf{r})\sim$ disc's modes: High energy containment*
- *$F^H(s,p)\sim$ gaussian function of mean value $\bar{p}=m/n_{\text{in}}kR$*
- *"Dynamical tunnelling" between regular and chaotic region; most probable exit zone near regular-chaotic transition: may trigger directional emission.*

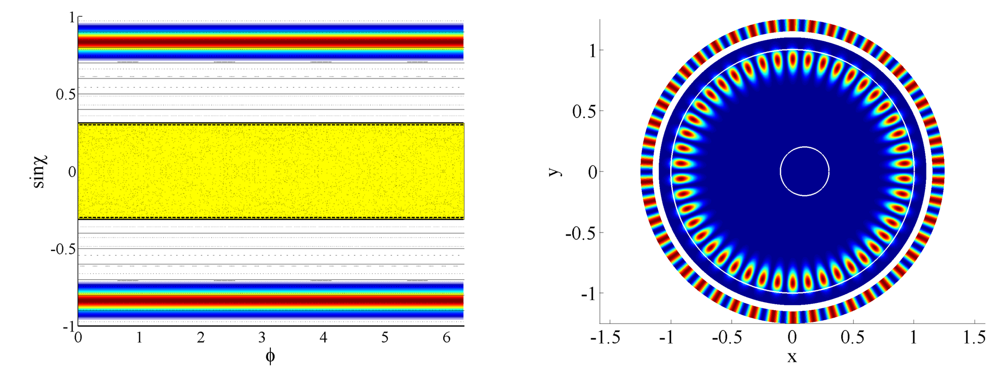


Fig. 8: ***$F^H(s,p)$ of mode $(27,1)$ for $k\sim8.594$, $d=0.1$, $R=1$, $a=0.2$, the chaotic region is embedded inside emission region: Right, isotropic emission in the far-field $r\to\infty$ (separated ring).***

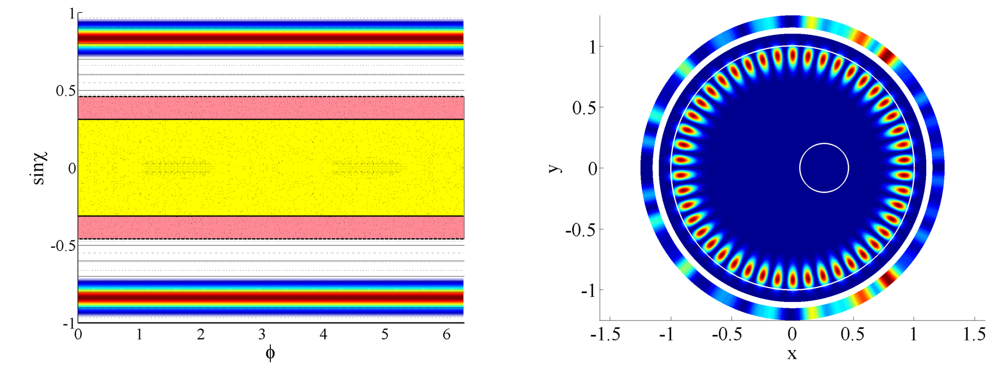


Fig. 9: ***For $d=0.26$, the chaotic region spreads out of emission region: Right, anisotropic emission in the far-field. Disc's mode $(27,1)$ and resonance level remain mostly unaffected.***

Harnessing the power flow: Parametric control

Figs. 8-9 show that it is possible to trigger anisotropic emission. We can now "control" the emission by modifying hole radius a while keeping $d+a=$ const. $(d+a=0.7)$:

- *Set of initial conditions around $|p|=(d+a)/R$*
- *$F^H(s,p)$ of mode $(27,1)$ is restricted to emission region.*

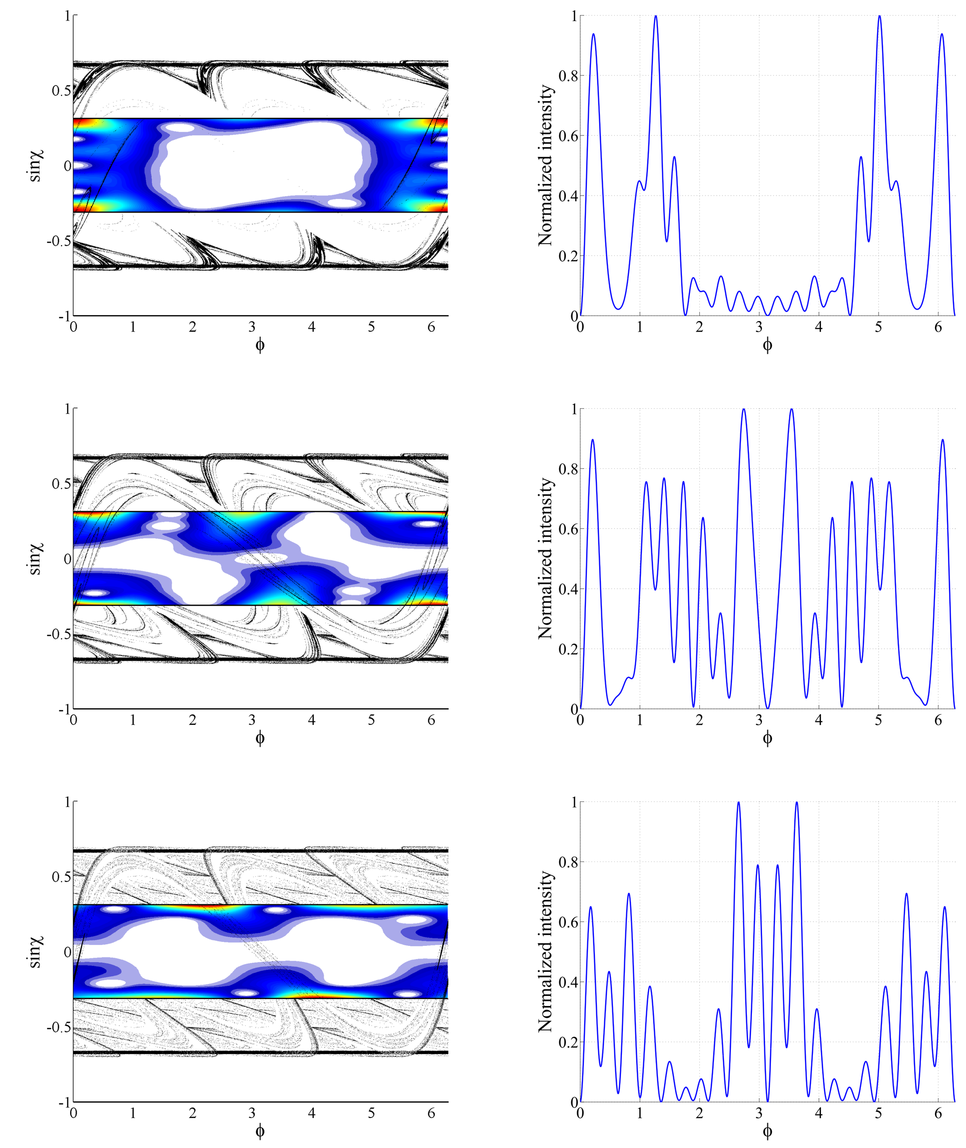


Fig. 10: ***Phase space with $F^H(s,p)$ and far-field emission for $a=0.3$ (top), $a=0.2$ (middle) and $a=0.1$ (bottom).***

- *Classical emission: Extension of unstable manifolds in emission region*
- *$F^H(s,p)$ follow unstable manifolds: Emission modified.*

Through the modification of phase space, we may then

- *Induce anisotropic emission of the disc's modes and*
- *Modify the far-field emission patterns*
- *While keeping high energy storage levels.*

[1] K. J. Vahala, Nature, **424**, 839-846 (2003).
[2] M. Hentschel *et al.*, Europhys. Lett., **62**, 636-642 (2003).
[3] S. Shinohara *et al.*, Phys. Rev. A, **74**, 033820-5 (2006).
[4] M. Hentschel *et al.*, Phys. Rev. E, **66**, 056207-13 (2002).

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