Optimization in optical systems revisited

Beyond genetic algorithms

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Optimization in physics

Optimization problems are ubiquitous in physics. Notable instances include

- Design of **integrated optical** devices
- Design of injectors and magnets in **accelerator** design
- Topological solitons in **nonlinear classical field theories**
- **Ising** models in condensed matter physics

Most real-life optimization problems cannot be solved analytically and are **NP-hard**. The most common approach is to use **metaheuristics**, algorithms based on empirical rules for exploring large **solution spaces**.

Two key concepts for metaheuristics

1. **Diversification**: Global exploration of the solution space in order to identify regions containing "fit" solutions
2. **Intensification**: More thorough investigation of "promising" solution regions [1].

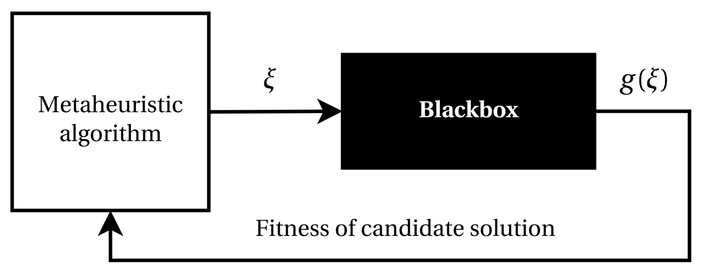

Fig. 1: Blackbox scenario for fitness function evaluation [1]

Laser beam shaping problem

Goal: To find a photonic lattice configuration which produces a scattered wavefunction that matches a desired profile in a given plane [2].

- Binary encoding
- Vertical symmetry
- Fitness function (scattered field) computed via **generalized Lorenz-Mie theory**.

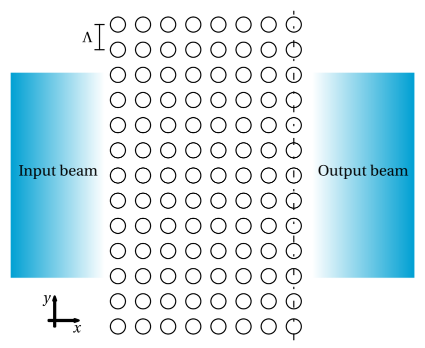

Fig. 2: Basic scatterer grid for the optimization problem. There are 2^{56} possible solutions

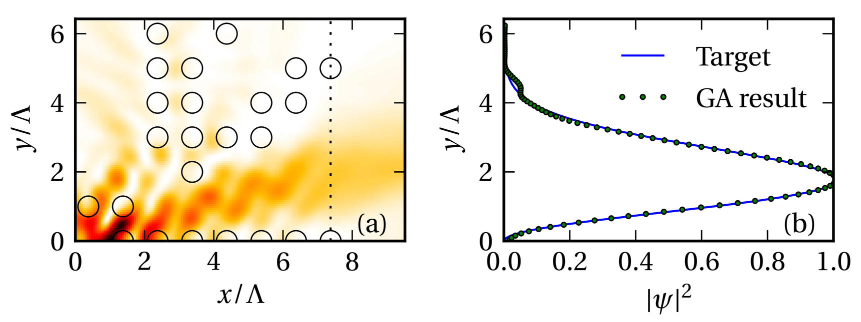

Fig. 3: Generating a $m = 1$ Hermite-Gauss beam using a genetically optimized photonic crystal lattice. Diameter of holes $D = 0.6\Lambda$, core index $n = 2.76$. The incident field is a TM-polarized gaussian beam with half-width $w_0 = 2.5\Lambda$ and incident wavenumber $k_0\Lambda = 1.76$ [3].

Genetic algorithm

Developed by J. Holland in the 1970's. Commonly used in photonics research, for instance **integrated waveguide** design [2].

- **Stochastic**, population-based, nature-inspired algorithm
- Memoryless method. The escape from local minima relies on random mutations
- Best suited for **diversification**. This stems from the population based nature of the algorithm
- 3 adjustable parameters to specify: Population size, **mutation** and **crossover** rates

Application to single-objective optimization

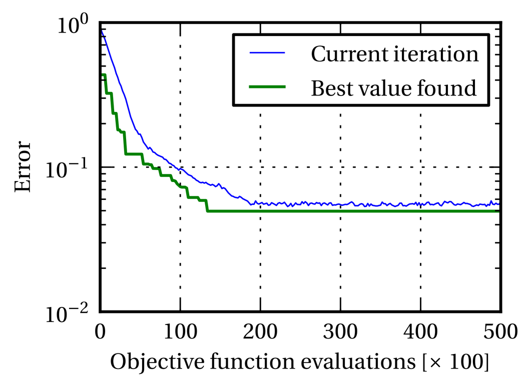

Fig. 4: Evolution diagram of the standard genetic algorithm (SGA) applied to the incoherent beam shaping problem. The algorithm exhibits rapid convergence to an "acceptable" solution, but tends to remain stuck in local minima (see definition of g_1 in bottom panel).

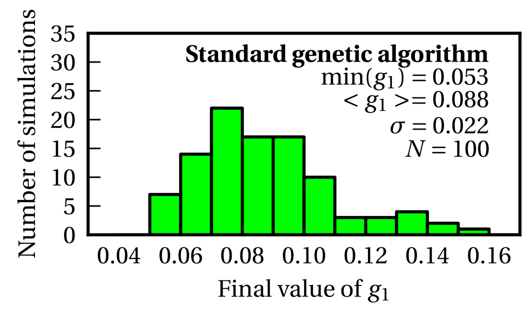

Fig. 5: Performance assessment of SGA. Each simulation represents 5000 generations, with an average of 60 objective function evaluations per generation.

Parallel tabu search

First proposed by F. Glover in the 1980's. More commonly used in **scheduling** and **networking** problems.

- **Deterministic**, local, non-nature inspired algorithm [1]
- Uses a **short-term memory** to escape from local minima
- Best suited for **intensification** of search. Parallel implementation allows to combine exploration and intensification. Initialization of solutions is the only random process
- Only 1 adjustable parameter: Number of entries in the **Tabu list**

Application to single-objective optimization

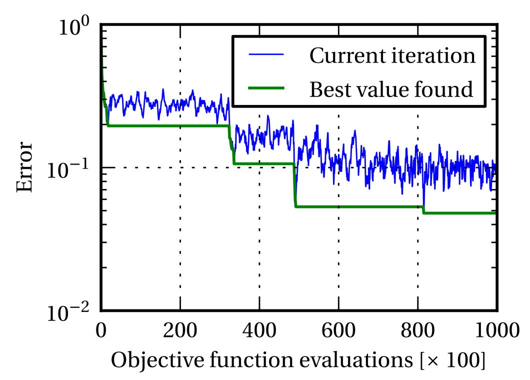

Fig. 6: Evolution diagram of one parallel tabu search (PTS) process applied to the incoherent beam shaping problem. Although initial convergence is slow, the short-term memory of the algorithm allows for "hill-climbing", thus avoiding getting stuck in local optima.

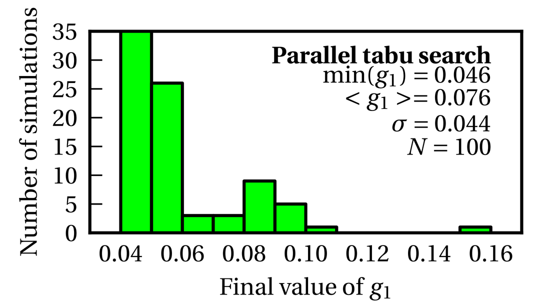

Fig. 7: Performance assessment of PTS. The algorithm exhibits better overall convergence, and finds some solutions inaccessible to the SGA. Each simulation represents 5000 iterations, with at most 56 objective function evaluations per iteration.

Multiobjective optimization (amplitude and phase control)

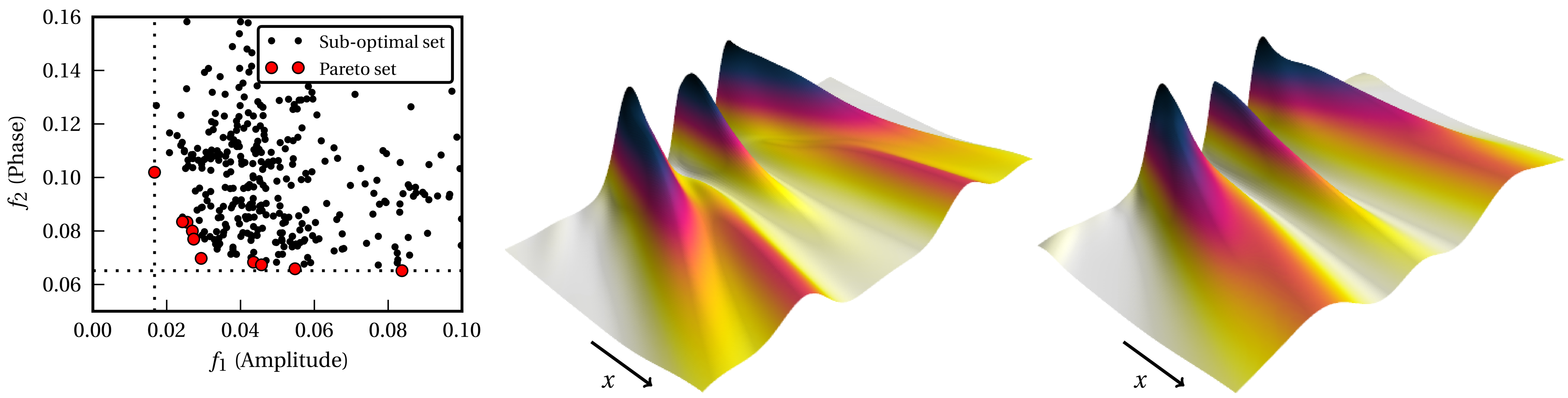

Fig. 8: Multiobjective optimization results obtained via PTS. **(Left)** Sampling of the Pareto front (set of optimal solutions satisfying both objectives) for the coherent beam shaping problem. The dotted lines indicate the best possible value for each of the two objectives. This sampling is achieved using an aggregate cost function. **(Center)** Optimized Hermite-Gauss beam profile at device output, with the best possible reproduction of the amplitude profile. **(Right)** Best possible trade-off between the two objectives. Since the phase is controlled, the Hermite-Gauss profile shape is preserved over a greater propagation distance. In other words, controlling both the amplitude and the phase allows for a greater field depth. This can be seen in the smaller number of ridges in the profile [4].

Aggregation method	Amplitude objective function	Phase objective function
$\min_{\xi\in\Xi}\sum_{i=1}^{p}\alpha_i f_i(\xi), \quad f_i = \frac{g_i}{g_i^{\max}}$	$g_1(\xi) = \frac{\int \left\lvert \lvert u(x_0,y)\rvert^2 - \lvert\bar{u}(x_0,y)\rvert^2\right\rvert dy}{\int \lvert\bar{u}(x_0,y)\rvert^2 dy}$	$g_2(\xi) = \frac{\int \left\lvert \mathrm{Im}[u(x_0,y)e^{-i\phi(x_0,0)}]\right\rvert^2 dy}{\int \lvert\bar{u}(x_0,y)\rvert^2 dy}$

Outlooks

Engineering of non-diffracting beams

Non-diffracting beams can be used in many applications, like **atom guiding** and **microscopy**. Various generation methods have been proposed.

- Axicon-shaped photonic crystals [H. Kurt, *J. Opt. Soc. Am. B* **26**, 981 (2009)]
- Phase plates optimized via GA [P. A. Sanchez-Serrano *et al.*, *Opt. Lett.* **37**, 5040 (2012)]
- Huygens' surfaces, composed of 2D arrays of polarizable particles [C. Pfeiffer and A. Grbic, *PRL* **110**, 197401+ (2013)]

Optimization of random laser action

Recent studies have shown that optimizing the pump shape allows control of **laser thresholds** and **emission directionality**. This optimization process implies the computation of a special kind of eigenstate, the **constant-flux state** [5].

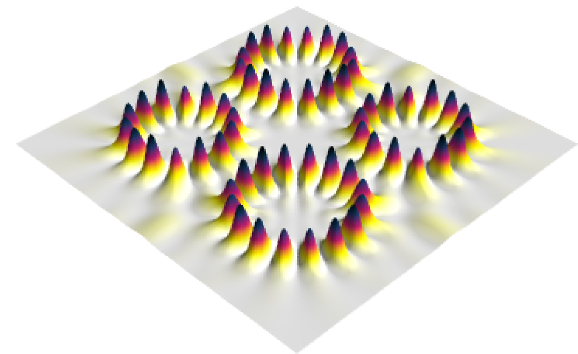

Fig. 9: Constant-flux state of an asymmetric photonic molecule composed of 4 dielectric atoms. Emission profile computed via **generalized Lorenz-Mie theory**. Constant-flux states are more physically meaningful than the usual quasi-bound states.

Summary

1. Since **parallel tabu search** combines search **diversification** and **intensification**, it outperforms the SGA in the case of our model problem of beam shaping.
2. The performance gain associated with PTS allows for **multi-objective optimization** in photonics design.
3. Optimization of **random lasers** and engineering **non diffracting beams** are potential applications of our algorithms in optics and photonics.

References

[1] E.-G. Talbi, Metaheuristics : From design to implementation (Wiley, 2009)
[2] A. Vukovic, P. Sewell, and T. M. Benson, *J. Opt. Soc. Am. A* **27**, 2156 (2010)
[3] **D. Gagnon, J. Dumont, L. J. Dubé, *J. Opt. Soc. Am. A* 29, 2673 (2012)**
[4] **D. Gagnon, J. Dumont, L. J. Dubé, *To appear in Opt. Lett.* (2013)**
[5] T. Hisch, M. Liertzer, D. Pogany, F. Mintert, and S. Rotter (2013), arxiv:1303.5292

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