# Analytical and computational approach to structure-function relationship in neural networks

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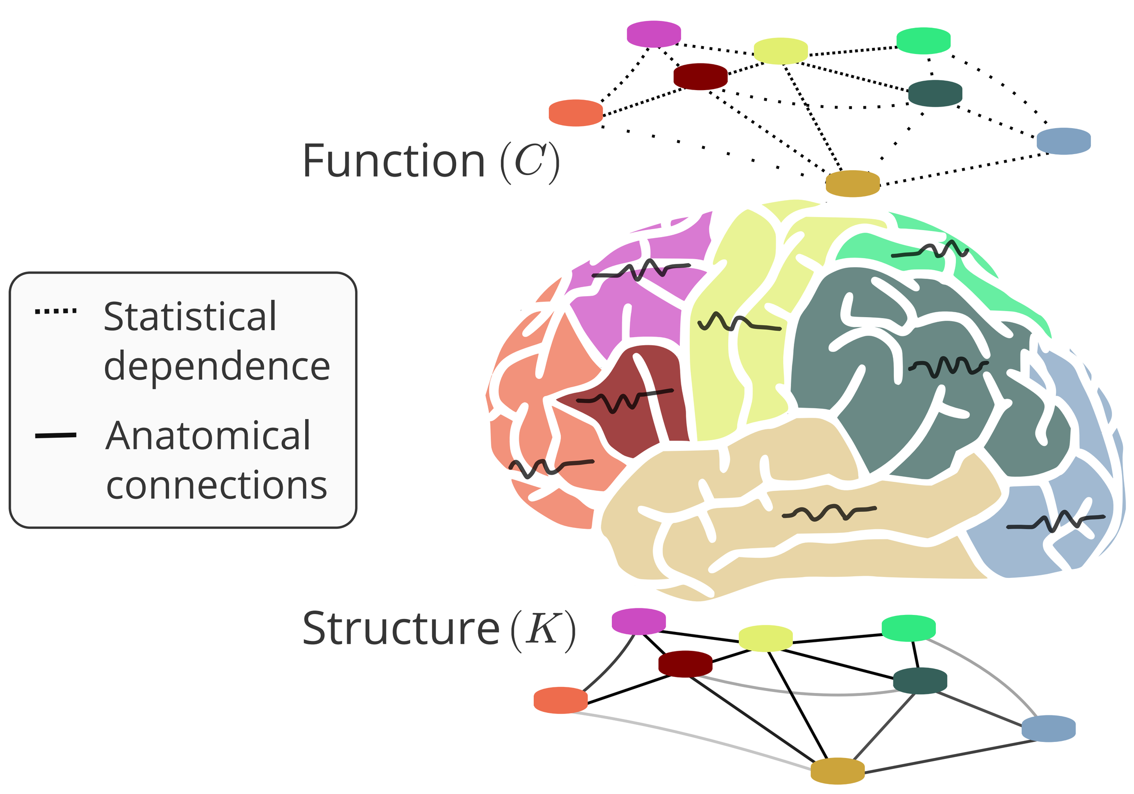
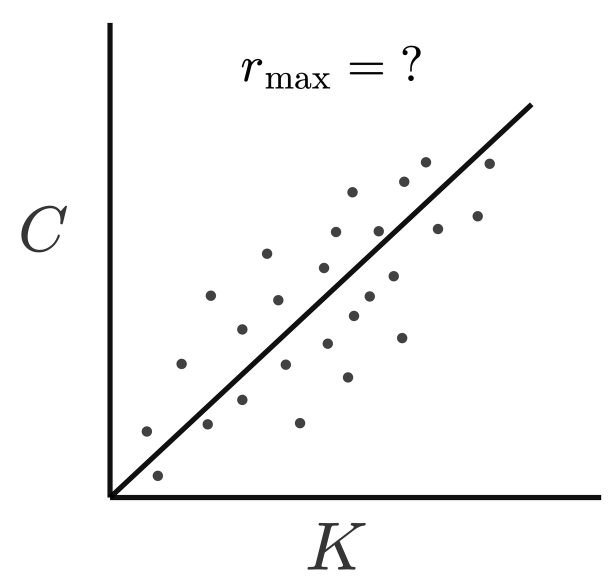
## Structure-function relationship

Functional connectivity (FC) describes statistical dependencies between the activity of neurons or groups of neurons [1]. Comparing FC with anatomical or structural connectivity (SC) has emerged as a promising avenue to study how brain structure supports function and how both change in disease or with cognition [2]. However, empirical studies across species and recording modalities have reported a wide range of FC-SC correspondence values, typically assessed using Pearson correlation [1,3]. Recent theoretical work further suggests that fundamental limits on the information shared among structure and dynamics may constrain our ability to relate FC to SC [4,5].




Interpreting structure-function relationships thus remains challenging and raises key questions: *how expected is a strong FC-SC correspondence,* and *how do different dynamical models shape the nature and strength of this relationship ?*

## Project goals and methods

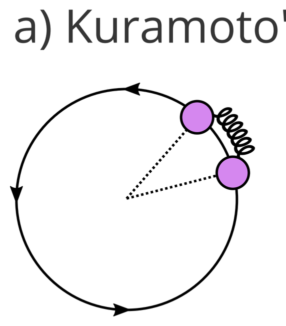
This project aims to identify specific dynamical regimes in which the structure-function correspondence is maximized, using established models of neural activity. It further seeks to determine how this relationship depends on the underlying dynamical model and to characterize it trough nonlinear measures that go beyond simple correlations.

**Computational framework.** We developped a computational framework (see SIMBA library) that systematically simulates the ordinary differential equations governing these models. By sweeping across relevant parameters, randomizing initial conditions, and monitoring observables from spectral graph theory and order parameters [6], we identified and characterized regimes exhibiting strong structure-function correspondence.

**Analytical investigations.** To evaluate how structural connectivity shapes functional dependencies, we derived approximate nonlinear relationships between the covariance of neural activity and the underlying SC across different models.

## Models of neural activity

a) Kuramoto's coupled oscillators:

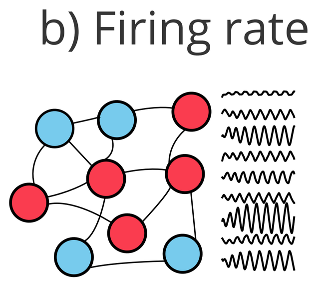


$$\frac{\mathrm{d}x_j}{\mathrm{d}t} = \omega_j + \hat{g}\sum_{l=1}^{N} K_{jl}\sin(x_l - x_j),$$

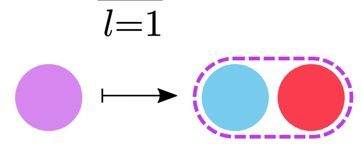
$\omega_j$ : natural frequency

$\hat{g} = \frac{g}{N}$ : reduced coupling

b) Firing rate with excitatory and inhibitory connections:



$$\frac{\mathrm{d}x_j}{\mathrm{d}t} = -x_j + \hat{g}\sum_{l=1}^{2N} W_{jl}\sigma(x_l) + \eta_j(t),$$



$\eta_j$ : noise process

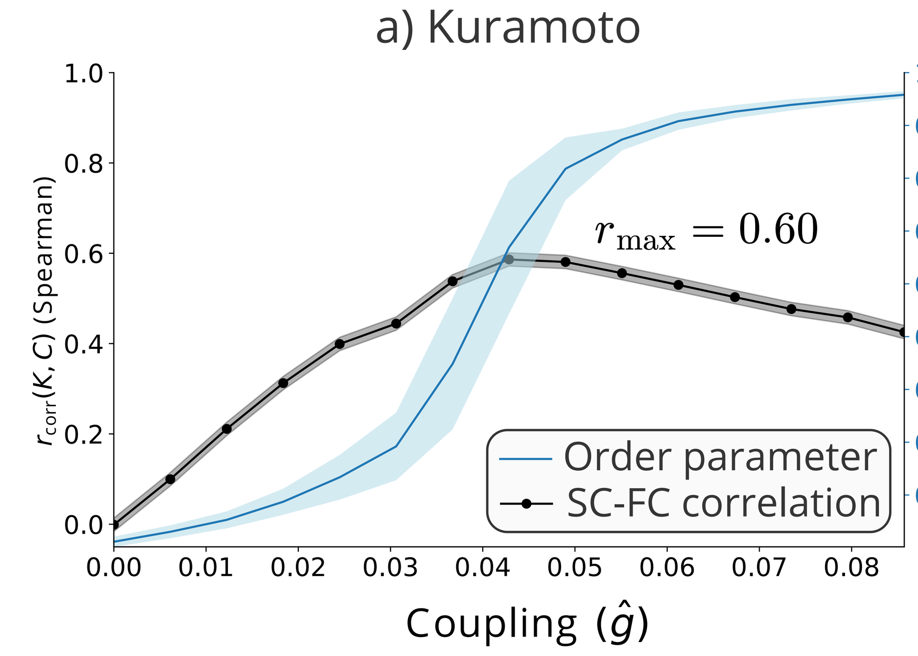
$\sigma(\cdot) = \frac{2}{\pi}\arctan(\cdot)$

$\hat{g} = \frac{g}{\sqrt{2N}}$

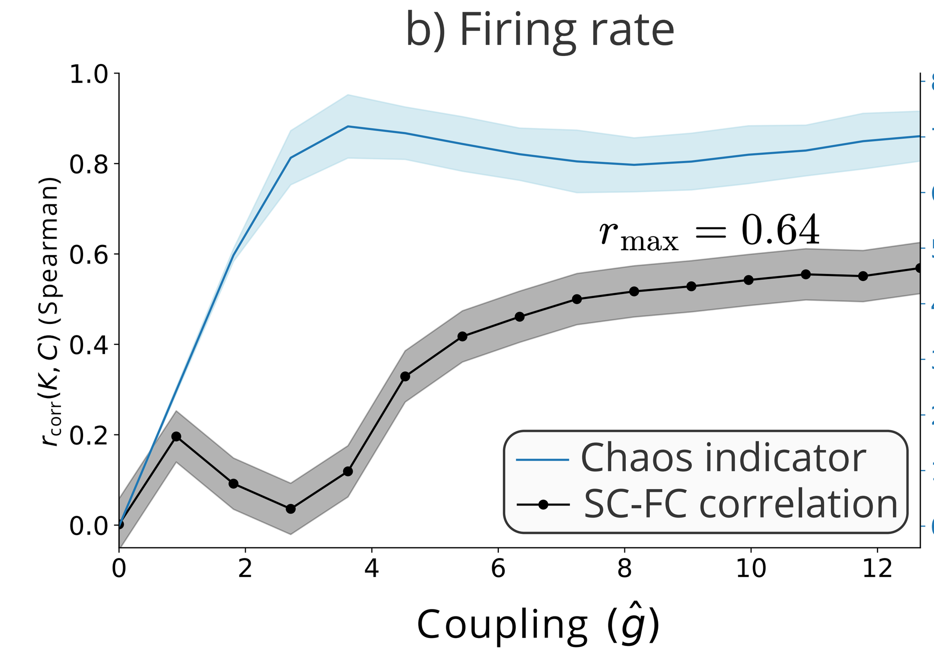
## Numerical simulations

***Example.*** SC-FC simulated on zebrafish mesoscale networks comprising 70 brain regions [6]

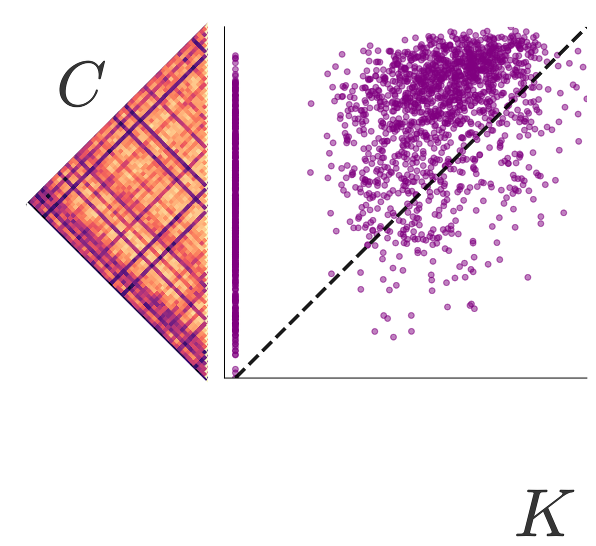
a) Kuramoto



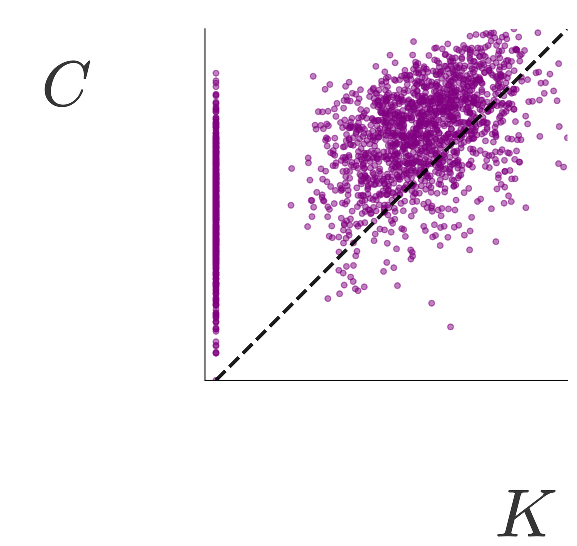

b) Firing rate




- $r_{\max}$ right before synchronization




- $r_{\max}$ reached for chaotic activity




## Analytical predictions

a) Kuramoto (low coupling):

*Covariance of sinus.* $C_{jk}(T) \approx \frac{1}{2}\mathrm{sinc}\left(\Delta_{jk}T\right) + \frac{\hat{g}^2}{8}\sum_{l=1}^{N}\frac{K_{jl}}{\Delta_{lj}}\frac{K_{kl}}{\Delta_{lk}}\bar{\delta}_{lj}\bar{\delta}_{lk}$

*Phase coherence.* $C_{jk}^2(T) \approx \mathrm{sinc}^2\left(\Delta_{jk}T/2\right)\left(1 + \hat{g}^2\sum_{l=1}^{N}\left(\frac{K_{jl}}{\Delta_{lj}}\bar{\delta}_{lj} - \frac{K_{kl}}{\Delta_{lk}}\bar{\delta}_{lk}\right)^2\right)$

$T$ : duration

$\Delta_{jk} = \omega_j - \omega_k$

$\bar{\delta}_{jk} = 1 - \delta_{jk}$

$\mathrm{sinc}(x) = \frac{\sin(x)}{x}$

b) Firing rate (covariance):

*Weak coupling.* $C \approx \alpha\left(W^{-1}\right)^2$ (Stochastic)

*Strong coupling.* $C \sim WW^\top$ (Mean-field) $C \approx \mathbb{I} + \frac{\epsilon^2}{4}\left(WW^\top\right)^{\circ -1}$ (Chaotic)

$\epsilon = \frac{1}{\hat{g}}$ : inverse coupling

$\alpha$ : noise amplitude

## Takeaways and future work

- Each model has its own optimal dynamical regimes for strong SC-FC correspondence.
  - — Kuramoto: FC correlates most with SC before transition to synchronization [7].
  - — Firing rate: FC globally aligns more with as SC as interneuronal coupling increases, except near the transition to chaos where this alignement reaches minimum.
- What's next:
  - — *How do such SC-FC relationships evolve according to bioplausible plasticity rules* [8] *?*
  - — *Within what structural limits do the closed-form expressions predict activity ?*

*Python Library.* **S**tructure **I**nfluence on **M**odels of **B**rain **A**ctivity, for simulation of many other models of neural activity.

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