# Firing rate distributions in plastic networks of spiking neurons

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Networks of spiking neurons have been widely used as models to represent neuronal activity in the brain. These models are reasonably realistic but they are also difficult to treat analytically. Mean-field theory has nevertheless proven to be successful as a method for deriving some of their statistical properties at equilibrium, such as the distribution of firing rates, either in fully homogeneous networks [1], networks with Erdös-Rényi connectivity [2] or networks which exhibit a large heterogeneity in their in- and out-degree distributions [3].

However, these models lack realism in the sense that they assume a fixed connectivity, whereas the connection strengths in brain networks evolve in time according to plasticity rules that depend on the neuronal activity. We have addressed this issue by extending the mean-field formalism to networks of leaky integrate-and-fire neurons with connections that are defined by a static binary scaffold but whose non-zero synaptic weights are prone to plastic, activity-dependent modulation. This provides a set of equations whose solution specifies the stationary firing rate and synaptic weight distributions.

The plasticity in our model is mediated by the introduction of spike traces, which are stochastic approximations to the individual firing rates. The temporal evolution of the trace associated to one neuron is controlled by the degradation speed of the trace (i.e., its “memory”) and by the mean temporal separation between consecutive spikes. These time scales jointly determine a shift from a regime characterized by highly noisy traces to a regime of accurate traces, and they in turn shape the system’s stationary distributions.

We show that the results are in good agreement with the distributions obtained by simulating the full spiking dynamics for quite general forms of plasticity functions. Our formalism sheds light on the interplay between the characteristic time scales of the neuronal and the plasticity dynamics, and can take into account the role of different types of neuronal communities (for example inhibitory and excitatory subnetworks). Overall, it offers a new perspective to explore and better understand the way in which plasticity shapes both activity and structure in neural networks.

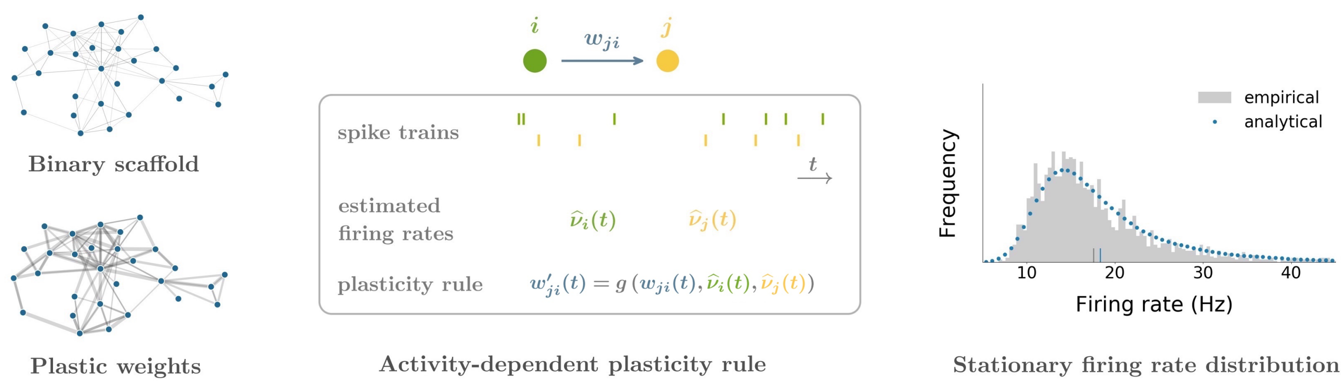


Figure 1: **Schematics of the model. Left:** The network has a fixed, binary scaffold on top of which the connection weights can evolve in time. **Center:** The plasticity rule as a way to modify the weight of a given connection $w_{ji}$ from neuron $i$ to neuron $j$. From the spiking times, stochastic variables $\widehat{\nu}_i(t)$, $\widehat{\nu}_j(t)$ (spikes traces) are updated and these provide an estimation to the individual firing rates. The weight $w_{ji}$ evolves as a function of the estimated rates. **Right:** The mean-field equations allow to analytically compute the distribution of firing rates in the stationary state.

[1] N. Brunel. Dynamics of sparsely connected networks of excitatory and inhibitory spiking neurons. *J Comput Neurosci*, 8(3):183–208, 2000.

[2] A. Roxin, N. Brunel, D. Hansel, G. Mongillo, and C. van Vreeswijk. On the distribution of firing rates in networks of cortical neurons. *J Neurosci*, 31(45):16217–16226, 2011.

[3] M. Vegué and A. Roxin. Firing rate distributions in spiking networks with heterogeneous connectivity. *Phys Rev E*, 100(2):022208, 2019.