Dimension reduction on heterogeneous networks

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The complexity inherent to large, non-linear dynamical systems of interacting units makes them very difficult to study. One of the big challenges of network science is finding ways to approximate these systems by systems of reduced dimension, which can be more tractable both analytically and computationally. Ideal reduced systems are those that are simple while preserving some of the key features of the systems they approximate. They can be used, for example, to predict tipping points, that is, values of the original system's parameters for which the overall behaviour of the system changes dramatically under small parameter perturbations. Despite some strategies have been proposed in the last years [1–5], a theoretical framework for dimension reduction of general systems of interacting units is still lacking. We have addressed this problem for dynamical systems whose units interact through a weighted, directed connectivity matrix. Our goal is to define a reduced set of observables which are linear combinations of the system's unit activities and a temporal evolution law for these observables that is able to capture essential properties of the complete system. Following the lines of previous work [3,4], we propose a two-step method for such a dimension reduction that takes into account the properties of the interaction matrix. First, units are partitioned into groups of similar connectivity properties. Each group is associated to one observable, which is a weighted average of the node activities within the group, so that the number of groups defines the dimension of the reduced system. Second, we provide a set of conditions that have to be fulfilled for these observables to properly represent the original system's behaviour, together with a method for approximately solving them. The result is a reduced interaction matrix and an approximate system of ODEs for the temporal evolution of the observables which is analogous in form to the original system. We show that the method works well for networks with a neat community structure. More interestingly, the reduced dynamics can also be used to predict the tipping points of systems that lack a community structure but which nevertheless exhibit a large degree of heterogeneity in their connectivity, even when the dimension of the reduced dynamics is much smaller than that of the original system. By offering a pragmatic and flexible way to regroup nodes, our formalism opens the way to a systematic comparison of the effect of various structural properties, both at the microscopic and mesoscopic levels, on the overall dynamics taking place on networks. It therefore allows us to identify the main structural driving forces guiding the evolution of dynamical processes on networks.

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