Time-dependent Spatial Growth of Complex Networks

Charles Murphy, Edward Laurence, Guillaume St-Onge, Jean-Gabriel Young, and Louis J. Dubé

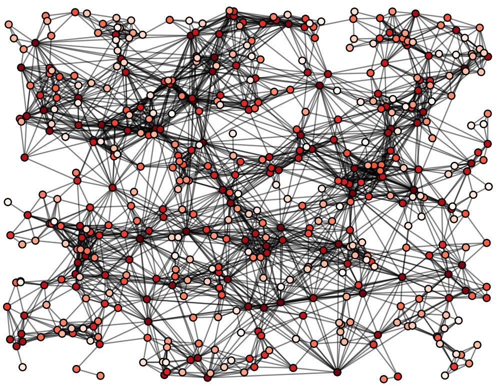
Département de Physique, de Génie Physique, et d'Optique, Université Laval, Québec, Qc, Canada

Real-world networks are often embedded in metric spaces constraining their connectivity. In a large variety of cases, from powergrids to road and neural networks, these spatial restrictions emerge from the minimization of the cost per edge as a function of the distance between the nodes. It has been widely studied in the literature under the paradigm of *random geometric graphs* [1], *spatial networks* and *spatially embedded networks* [2]. However, these networks are often treated as static, in contrast to real complex systems for which evolution is ever present.

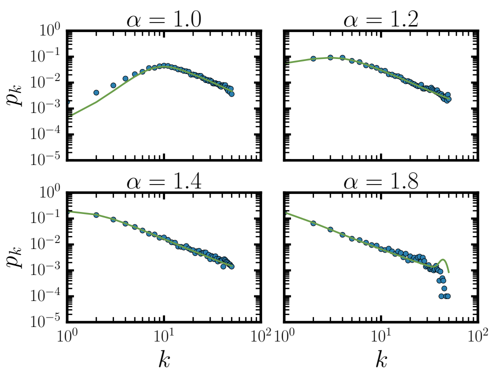
To address this issue, we have developed a spatial network growth model for which the connection probability depends on the geometric distance between the nodes and on the time of growth event. From this simple model emerge naturally some desirable network attributes, such as high clustering, formation of rich-clubs, hierarchy and modularity.

Our initial study focuses on a simple version of our general model, namely a growing random geometric graphs with a *time-dependent* distance threshold for connection $R(t) \propto (t + \tau)^{-\alpha}$. At each growth event, a new node is assigned a random position and is connected to his neighbors if the distance is less than $R(t)$. We obtain a number of theoretical results for this growth process and, in particular, we validate the analytical solution of the degree distribution with Monte Carlo simulations (Fig. 1). Our growth model offers a flexible framework to understand the structure of a variety of spatial and evolving complex systems with applications ranging from powergrids to connectomics [3].

[1] Dall J., and Christensen M., Random geometric graphs, *Phys. Rev. E* **66**, 016121, (2002).
[2] Barnett L., Di Paolo E., and Bullock S., Spatially embedded random networks, *Phys. Rev. E* **76**, 056115, (2007).
[3] Bullmore E., and Sporns O., The economy of brain network organization, *Nat. Rev. Neurosci.* **13**, 186, (2009).



(a)



(b)

Figure 1: (a) Example of a time-dependent random geometric graph with $R(t) = \tau^{\alpha}(t + \tau)^{-\alpha}$, $\tau = 5$, $\alpha = 0.6$ and the number of nodes $N = 500$. The age of the nodes is colour-coded, from red to white: red nodes are the oldest whereas the white nodes have been created the latest in the growth process. Older nodes can make longer connections than younger nodes and have a higher probability to be connected with each other, thus forming a strong rich-club. (b) Degree distribution of the growing random geometric graphs for different values of α with fixed $\tau = 50$ and $N = 1000$. The blue dots are averaged results of 10 Monte Carlo simulations while the green line corresponds to the analytical solutions. The growth towards a power law behaviour is evident as well as a finite-size cut-off for the highest α.