Bond and site percolation on clustered and correlated random graphs

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We present a general and exact mathematical framework to study bond and site percolation on clustered and correlated random graphs. This model uses various kind of arbitrary motifs to simulate clustering, and introduces different node types to account for correlations in the way nodes are connected to one another.

Our framework allows, among other things, the calculation of the percolation threshold, the size and probability of an extensive (giant) component, as well as the size and composition distribution of intensive (small) components. The calculation is performed in two stages. The bond/site percolation on each type of motifs (yielding a size distribution of components) is first solved exactly using a set of iterative equations. This restores the treelike structure of the graphs, and permits, in a second step, the use of probability generating functions to obtain the abovementioned properties.

We argue that the generality of our framework makes it one of the most versatile theoretical tools to study percolation on random graphs. As an illustration, we demonstrate the conditions under which there exists a bijective relationship between bond and site percolation. By providing a counterexample, we show that a recent criterion introduced to distinguish *strong and weak clustering regimes* [1] may be a necessary condition but is however not a sufficient one. We finally use our formalism to identify a new regime in which two extensive components coexist (see figure below) in the context of *network observability transition* discussed in [2].

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[1] M. Á. Serrano and M. Boguñá, Clustering in complex networks. II. Percolation properties, *Phys. Rev. E*, 74:056115, 2006.
[2] Y. Yang, J. Wang, and A. E. Motter, Network Observability Transitions, *Phys. Rev. Lett.*, 109:258701, 2012.

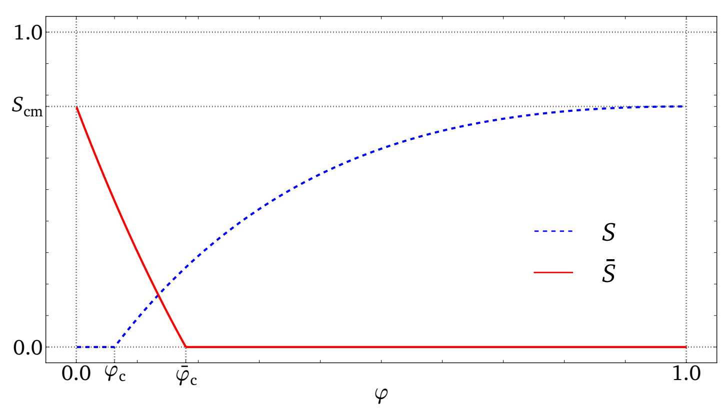

Figure 1: Relative size of the *giant observable component* (S, blue, composed of observable nodes and their first neighbours) and of the *giant non-observable component* ($\bar{S}$, red, composed of non-observable nodes) as a function of the node observability probability φ. The two threshold values, φ_c and $\bar{\varphi}_c$, are shown, as well as the relative size of the giant component of the corresponding Configuration Model, S_{cm}.