Exact(?) SIR dynamics on networks of finite size

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Both stochasticity and finite size have important effects on the invasion dynamics of epidemiological systems. Indeed, randomness may cause over-threshold epidemics to die out in the early stage of the infection while epidemics reaching a sizeable fraction of a finite network are slowed down before going extinct. We consider the continuous time dynamics of a Susceptible-Infectious-Removed (SIR) epidemic model on finite-size networks with heterogeneous degree distributions. We obtain a high-dimensional differential equation describing the time evolution of the probability distribution of the states of the system. This differential equation appears to be exact "by design" for networks with a prescribed degree sequence where repeated links and self-loops are allowed. Analytical calculations for specific small networks and numerical simulations of moderate-size networks support this exactness hypothesis. Since repeated links and self-loops have negligible impact on the dynamics of larger networks, the differential equation should also be a good approximation in large networks when those patterns are forbidden. The high-dimensionality of the system poses challenges to numerical simulation of large systems; analytical approaches and approximations are currently studied.