Bursty exposure on higher-order networks leads to nonlinear infection kernels

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Three properties of human dynamics and disease transmissions are often overlooked in standard epidemic models: the hypergraph structure of contacts, the burstiness of human behavior, and the complex nonlinear relationship between the exposure to infected contacts and the risk of infection.

In this work, we combine these three properties in a hypergraph contagion model, with hyperedges representing environments where individuals can interact, and where a minimal number of interactions with infected individuals are needed to contract the disease. The central result we obtain is that bursty exposure, modeled by a power-law distribution of participation time to environments, can induce a nonlinear relationship between the number of infected participants and the probability to become infected. We then demonstrate how conventional epidemic wisdom can break down with the emergence of discontinuous transitions, super-exponential spread, and regimes of hysteresis.

On a theoretical level, this work formally provides a connection between complex contagions based on nonlinear infection kernels and threshold models. This also allows for a deeper understanding of how higher-order interactions and burstiness affect epidemic spreading. On the epidemiological level, our results challenge a key assumption of most epidemic models and ask: Why assume a linear relationship between the number of infectious contacts and the risk of infection?

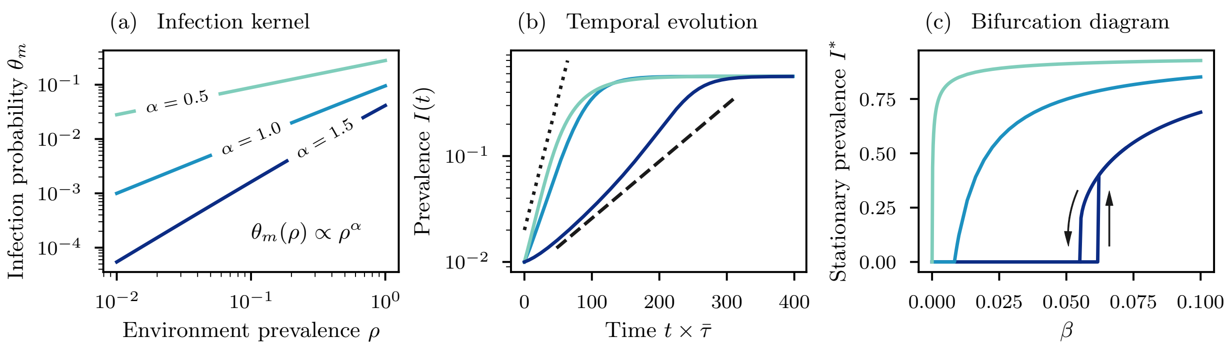

Figure 1: Bursty exposure induces contagions with nonlinear infection kernels. We assume that an individual must have ($K = 2$) interactions with infected individuals in a hyperedge (environment) to become infected. Each individual participates a time τ to a hyperedge, distributed according to $P(\tau) \propto \tau^{-\alpha-1}$ with $\tau \in [1, \infty)$. (a) Effective infection probability per participation to a hyperedge of any size m. The infection probability has a power-law scaling $\theta_m(\rho) \propto \rho^\alpha$ if $K \geq \alpha$. (b)-(c) We study the consequences of nonlinear infection kernels for a SIS-type contagion dynamics on hypergraphs. The average number of interactions in an environment is proportional to β. (b) Supra-linear kernel $\alpha > 1$ leads to a super-exponential growth for the global prevalence $I(t)$. We adjusted β to obtain similar prevalence levels. $\bar{\tau}$ is the median participation time associated with $P(\tau)$. (c) The bifurcation diagram in the stationary state ($t \to \infty$) can be continuous or discontinuous with a bistable regime. Sub-linear and linear kernels $\alpha \leq 1$ lead to a continuous phase transition, and the epidemic threshold β_c vanishes for $\alpha \to 0$. Supra-linear kernels $\alpha > 1$ can lead to a discontinuous phase transition with a bistable regime.