## Dimension reduction of high-dimensional dynamics on networks with adaptation

Adaptation is a defining property of complex systems. In the brain, one typical form of adaptation is synaptic plasticity: the synapses strengthen or weaken over time in response to neuronal activity. However, predicting the impact of adaptation on the overall evolution of neuronal networks remains a challenging problem, in part due to the high-dimensionality of the dynamics that governs both plasticity and neuronal activity. Based on our previous work [Thibeault et al., Phys. Rev. Research, 2020], we introduce a new dimension reduction framework that systematically yields a reduced adaptative dynamics from a high-dimensional adaptative dynamics. The reduced dynamics accurately describes the Wilson-Cowan dynamics for various adaptation rules: Hebb's rule, Oja's rule, and the biologically plausible Bienenstock-Cooper-Monroe's (BCM) rule [Cooper et al., Nat. Rev. Neurosci., 2012]. The Wilson-Cowan dynamics with the BCM rule exhibits rich bifurcation phenomena that are well predicted by the reduced adaptative dynamics. For instance, we observe the emergence of surprising nonlinear oscillations in the firing rate and the synaptic strength which appear through a supercritical Hopf bifurcation. Our framework is flexible: we can tune the number of dimensions of the reduced system to improve accuracy. It also unlocks the possibility to perform dynamical analysis for large real networks, which paves the way towards a better understanding of emergent phenomena in the brain.