Dimension reduction of high-dimensional dynamics on networks with adaptation

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Adaptation is a defining property of complex systems. It is characterized by changes in the structure and the dynamics of the system according to its environment and its own behavior [1]. For instance, neurons in the brain are connected by synapses that increase or decrease in strength as a response to neuronal activity. Synaptic plasticity is known to be an essential mechanism of learning and memory, but its precise role and its global influence on brain activity remain unclear [2]. From a theoretical standpoint, the impact of adaptation on the neuronal dynamics' equilibrium points is hard to predict. This is partly due to the high dimensionality of the dynamical system describing adaptative neuronal networks. Indeed, in addition to the ordinary differential equations (ODEs) describing the activity of all neurons (nodes), there is another large set of ODEs governing the evolution of the strength (weight) of all synapses (edges).

Based on our previous works [3-4], we introduce a new dimension reduction framework that systematically yields a low-dimensional (reduced) adaptative dynamics from a high-dimensional (complete) adaptative dynamics. The reduced dynamics accurately describes the complete Wilson-Cowan dynamics with three different adaptation rules: Hebb's rule, Oja's rule, and even the biologically plausible Bienenstock-Cooper-Monroe's (BCM) rule [5]. The Wilson-Cowan dynamics with the BCM rule exhibits rich bifurcation phenomena that are well predicted by the reduced adaptative dynamics [FIG. 1]. For instance, our dimension reduction framework captures the emergence of surprising nonlinear oscillations in the firing rate and the synaptic strength which appear through a supercritical Hopf bifurcation.

Our framework is flexible: we can tune the number of observables as we want to describe adaptative networks with more modules and heterogeneity. It also unlocks the possibility to perform dynamical analysis on large real networks which paves the way towards a better understanding of emergent phenomena in the plastic brain.

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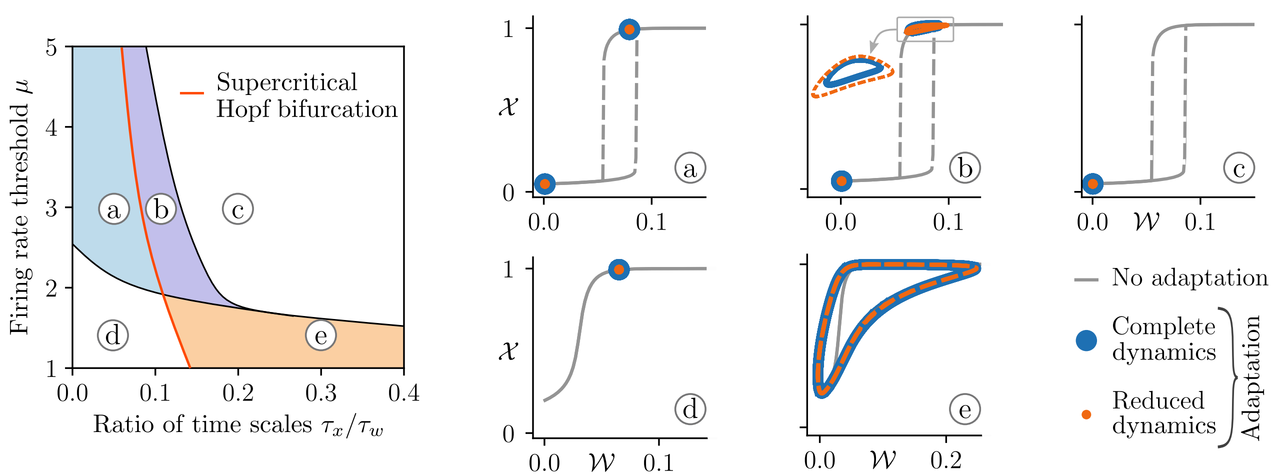

FIG. 1. Bifurcation diagram of the Wilson-Cowan dynamics with the BCM rule (10 200 ODEs) obtained from its reduced dynamics (3 ODEs). The firing rate threshold μ is the midpoint of the sigmoidal activation function of the Wilson-Cowan dynamics and τ_x is the time scale of the activity while τ_w is the one for the weights. Figures a-e illustrate the equilibrium points of the mean global activity observable $\mathcal{X}$ vs. the mean global weight observable $\mathcal{W}$ when there is no adaptation and when there is adaptation for the complete and reduced dynamics. These figures show that the reduced dynamics predicts accurately the equilibrium points in every region of the bifurcation diagram. We choose a weighted Erdős-Rényi network with $N = 100$ and $p = 0.5$ as an initial condition for the synaptic dynamics.